Integration

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Integration
Booklet	Question Paper

Time Allowed: **108 minutes**

Score: **/90**

Percentage: **/100**

Grade Boundaries:

1 **(a)** Show that $\int_0^2 \frac{x}{x^2+5}\,\mathrm{d}x = \ln\left(\frac{3}{\sqrt{5}}\right)$. [4]

(b) Find $\int x\sqrt{x-2}\,\mathrm{d}x$. [4]

2 **(i)** Find $\int (3x^2 - 4x + 8)\,\mathrm{d}x$. [3]

(ii) Hence find $\int_1^3 (3x^2 - 4x + 8)\,\mathrm{d}x$. [2]

3 **(i)** Given that $\frac{2x+11}{(2x+1)(x+3)} \equiv \frac{A}{2x+1} + \frac{B}{x+3}$, find the values of the constants A and B. [4]

(ii) Hence show that $\int_0^2 \frac{2x+11}{(2x+1)(x+3)}\,\mathrm{d}x = \ln 15$. [5]

4 **(i)** Use integration by parts to show that $\int \ln x\,\mathrm{d}x = x\ln x - x + c$. [2]

(ii) Find

(a) $\int (\ln x)^2\,\mathrm{d}x$, [4]

(b) $\int \frac{\ln(\ln x)}{x}\,\mathrm{d}x$. [5]

5 Find the exact value of $\int_0^1 (\mathrm{e}^x - x)\,\mathrm{d}x$. [4]

6 **(i)** **(a)** Find $\int \frac{e^x}{1+e^x}\,dx$. [2]

(b) Hence evaluate $\int_0^{\ln 3} \frac{e^x}{1+e^x}\,dx$, giving your answer in the form $\ln k$, where k is an integer. [3]

(ii) **(a)** Using the substitution $u = 1 + e^x$, find $\int \left(\frac{e^x}{1+e^x}\right)^2 dx$. [5]

(b) Hence find the exact volume of the solid of revolution generated when the curve given by $y = \frac{e^x}{1+e^x}$, between $x = -\ln 3$ and $x = \ln 3$, is rotated through 2π radians about the x-axis. [2]

7 A circle has equation $x^2 + y^2 = 16$. Find the volume generated when the region in the first quadrant which is bounded by the circle and the lines $x = 1$ and $x = 2$ is rotated through 2π radians about the x-axis. [5]

8 Use integration by parts to find $\int x \sin 3x\,dx$. [5]

9 **(i)** Using the substitution $u = x^2$, or otherwise, find the numerical value of

$$\int_0^{\sqrt{\ln 4}} xe^{-\frac{1}{2}x^2}\,dx.$$ [4]

(ii) Determine the exact coordinates of the stationary points of the curve $y = xe^{-\frac{1}{2}x^2}$. [4]

10 Using the substitution $u = 1 + \sqrt{x}$, or otherwise, find $\int \frac{1}{1+\sqrt{x}}\,dx$ giving your answer in terms of x. [5]

11 Find the exact value of

$$\int_1^4 \left(10x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\right) \mathrm{d}x. \qquad [3]$$

12 **(i)** Show that

$$\int_1^a x^n \ln x \, \mathrm{d}x = \frac{a^{n+1}}{(n+1)^2}\left((n+1)\ln a - 1\right) + \frac{1}{(n+1)^2},$$

where $n \neq -1$ and $a > 1$. [6]

(ii) **(a)** Determine the x-coordinate of the point of intersection of the curves $y = x^2 \ln x$ and $y = x \ln 2^x$, where $x > 0$. [2]

(b) Find the exact value of the area of the region enclosed between these two curves, the line $x = 1$ and their point of intersection. Express your answer in the form $b + c \ln 2$, where b and c are rational. [4]

(iii) The curve $y = (x^3 \ln x)^{0.5}$, for $1 < x < \mathrm{e}$, is rotated through 2π radians about the x-axis. Determine the value of the resulting volume of revolution, giving your answer correct to 4 significant figures. [3]