Chapter 3 Trigonometry

May/June 2002

1 Prove the identity

$$\cot\theta - \tan\theta \equiv 2\cot 2\theta.$$ [3]

Oct/Nov 2002

5 **(i)** Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$4\sin\theta - 3\cos\theta = 2,$$

giving all values of θ such that $0° < \theta < 360°$, [4]

(iii) write down the greatest value of $\dfrac{1}{4\sin\theta - 3\cos\theta + 6}$. [1]

May/June 2003

1 **(i)** Show that the equation

$$\sin(x - 60°) - \cos(30° - x) = 1$$

can be written in the form $\cos x = k$, where k is a constant. [2]

(ii) Hence solve the equation, for $0° < x < 180°$. [2]

10 **(i)** Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x.$$ [3]

(ii) Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \cot x \, dx = \frac{1}{2}\ln 2$. [3]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \operatorname{cosec} 2x \, dx$, giving your answer in the form $a \ln b$. [4]

Oct/Nov 2003

3 Solve the equation

$$\cos\theta + 3\cos 2\theta = 2,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

May/June 2004

1 Sketch the graph of $y = \sec x$, for $0 \leqslant x \leqslant 2\pi$. [3]

5 **(i)** Prove the identity

$$\sin^2\theta\cos^2\theta \equiv \tfrac{1}{8}(1-\cos 4\theta).$$ [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2\theta\cos^2\theta\,\mathrm{d}\theta.$$ [3]

Oct/Nov 2004

4 **(i)** Show that the equation

$$\tan(45^\circ + x) = 2\tan(45^\circ - x)$$

can be written in the form

$$\tan^2 x - 6\tan x + 1 = 0.$$ [4]

(ii) Hence solve the equation $\tan(45^\circ + x) = 2\tan(45^\circ - x)$, for $0^\circ < x < 90^\circ$. [3]

May/June 2005

6 **(i)** Prove the identity

$$\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3.$$ [4]

(ii) Hence solve the equation

$$\cos 4\theta + 4\cos 2\theta = 2,$$

for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

Oct/Nov 2005

5 By expressing $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, solve the equation

$$8\sin\theta - 6\cos\theta = 7,$$

for $0^\circ \leqslant \theta \leqslant 360^\circ$. [7]

May/June 2006

4 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

Oct/Nov 2006

2 Solve the equation

$$\tan x \tan 2x = 1,$$

giving all solutions in the interval $0^\circ < x < 180^\circ$. [4]

May/June 2007

5 **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + (\sqrt{3})\sin\theta)^2}\,d\theta = \frac{1}{\sqrt{3}}$. [4]

Oct/Nov 2007

5 **(i)** Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0. \quad [3]$$

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

May/June 2008

4 **(i)** Show that the equation $\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0. \quad [4]$$

(ii) Hence, or otherwise, solve the equation

$$\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta),$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

Oct/Nov 2008

6 **(i)** Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\sin 2\theta + 12\cos 2\theta = 11,$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

May/June 2009

3 **(i)** Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot\theta$. [3]

(ii) Hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [2]

Oct/Nov 2009/31

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta + 3 \equiv 8\sin^4\theta$. [4]

(ii) Using this result find, in simplified form, the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin^4\theta \, d\theta.$$ [4]

Oct/Nov 2009/32

4 The angles α and β lie in the interval $0° < x < 180°$, and are such that

$$\tan\alpha = 2\tan\beta \quad \text{and} \quad \tan(\alpha + \beta) = 3.$$

Find the possible values of α and β. [6]

May/June 2010/31

2 Solve the equation

$$\sin\theta = 2\cos 2\theta + 1,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

4 **(i)** Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\tfrac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x.$$ [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x \, dx = \tfrac{1}{8}\sqrt{3}.$$ [3]

May/June 2010/32

3 It is given that $\cos a = \frac{3}{5}$, where $0° < a < 90°$. Showing your working and without using a calculator to evaluate a,

(i) find the exact value of $\sin(a - 30°)$, [3]

(ii) find the exact value of $\tan 2a$, and hence find the exact value of $\tan 3a$. [4]

May/June 2010/33

3 Solve the equation

$$\tan(45° - x) = 2\tan x,$$

giving all solutions in the interval $0° < x < 180°$. [5]

7 **(i)** Prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [4]

(ii) Using this result, find the exact value of

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3\theta \, d\theta.$$ [4]