Discrete Random Variables

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Statistics- Discrete Random Variables
Booklet	Question Paper

Time Allowed: **76   minutes**

Score: **/63**

Percentage: **/100**

Grade Boundaries:

1   A survey into left-handedness found that 13% of the population of the world are left-handed.

    (i) State the assumptions necessary for it to be appropriate to model the number of left-handed children in a class of 20 children using the binomial distribution B(20, 0.13).   [2]

    (ii) Assuming that this binomial model is appropriate, calculate the probability that fewer than 13% of the 20 children are left-handed.   [4]

2   James plays an arcade game. Each time he plays, he puts a £1 coin in the slot to start the game. The possible outcomes of each game are as follows:

    James loses the game with a probability of 0.7 and the machine pays out nothing,
    James draws the game with a probability of 0.25 and the machine pays out a £1 coin,
    James wins the game with a probability of 0.05 and the machine pays out ten £1 coins.

The outcomes can be modelled by a random variable X representing the number of £1 coins gained at the end of a game.

    (i) Construct a probability distribution table for X.   [2]

    (ii) Show that E(X) = –0.25 and find Var(X).   [4]

James starts off with 10 £1 coins and decides to play exactly 10 games.

    (iii) Find the expected number of £1 coins that James will have at the end of his 10 games.   [2]

    (iv) Find the probability that after his 10 games James will have at least 10 £1 coins left.   [3]

3   A discrete random variable X has the following probability distribution.

x	1	2	n	7
P($X = x$)	0.4	0.3	p	0.1

    (i) Write down the value of p.   [1]

    (ii) Given that E(X) = 2.5, f nd n.   [2]

    (iii) Find Var(X).   [3]

4   In a certain country 40% of the population have brown eyes. A random sample of 20 people is chosen from that population.

    (i) Find the expected number of people in the sample who have brown eyes.   [1]

    (ii) Find the probability that there are exactly 8 people with brown eyes in the sample.   [3]

    (iii) Find the probability that there are at least 8 people with brown eyes in the sample.   [2]

5   A bag contains four black balls and one white ball. A man chooses a ball at random. If it is a black ball, he replaces it and chooses another at random. If he chooses the white ball, he stops.

    (i) Name the probability distribution which models this situation.   [1]

    (ii) Calculate the probability that he will make exactly three attempts before he stops.   [2]

    (iii) Calculate the probability that he will make fewer than three attempts before he stops.   [2]

6   In one department of a firm four employees are selected for promotion from a staff of eighteen.

    (i) In how many ways can four employees be selected?   [2]

      It is known that throughout the f rm 5% of those selected for promotion decline it.

    (ii) If 100 employees are randomly selected for promotion in the f rm, calculate the number expected to decline promotion.   [1]

    (iii) If 20 employees are selected at random for promotion, use the binomial distribution to f nd the probability that fewer than f ve employees will decline promotion.   [3]

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James draws the game with a probability of 0.25 and the machine pays out a £1 coin,

James wins the game with a probability of 0.05 and the machine pays out ten £1 coins.

The outcomes can be modelled by a random variable X representing the number of £1 coins gained at the end of a game.

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 (ii) Show that E(X) = −0.25 and f nd Var(X).   [4]

James starts off with 10 £1 coins and decides to play exactly 10 games.

(iii) Find the expected number of £1 coins that James will have at the end of his 10 games.   [2]

(iv) Find the probability that after his 10 games James will have at least 10 £1 coins left.   [3]

8 (a) In a game show contestants are asked up to five questions in succession to qualify for the next round. An incorrect answer eliminates a contestant from the game show.

Let X denote the number of questions correctly answered by a contestant. The probability distribution of X is given below.

x	0	1	2	3	4	5
P($X = x$)	0.30	0.25	0.20	0.16	0.06	0.03

    (i) Find the expected number of correctly answered questions and the variance of the distribution.   [3]

    (ii) Find the probability that a randomly selected contestant will correctly answer 3 or more questions.   [1]

    (iii) Each show had two contestants. Find the probability that both the contestants will correctly answer at least one question.   [2]

(b) In a promotion, a newspaper included a token in every copy of the newspaper. A proportion, 0.002, are winning tokens and occur randomly. A reader keeps buying copies of the newspaper until he buys one with a winning token and then stops.

Let Y denote the number of copies bought.

    (i) Explain briefly why this situation may be modelled by a geometric distribution and write down a formula for P($Y = y$).   [2]

    (ii) Find the probability that the reader gets a winning token with the twentieth copy bought.   [2]

    (iii) Find the probability that the reader will not have to buy more than three copies in order to get a winning token.   [2]