Complex Numbers

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Complex Numbers
Booklet	Question Paper

Time Allowed: **71 minutes**

Score: **/59**

Percentage: **/100**

Grade Boundaries:

1 The complex number $3 - 4\mathrm{i}$ is denoted by z. Giving your answers in the form $x + \mathrm{i}y$, and showing clearly how you obtain them, find

(i) $2z + z^*$, [2]

(ii) $\dfrac{5}{z}$. [2]

(iii) Show z and z^* on an Argand diagram. [2]

2 A root of the equation $z^2 + pz + q = 0$ is $3 + \mathrm{i}$, where p and q are real. Write down the other root of the equation and hence calculate the values of p and q. [4]

3 **(i)** Express $z^4 + 3z^2 - 4$ in the form $(z^2 + a)(z^2 + b)$ where a and b are real constants to be found. [2]

(ii) Hence draw an Argand diagram showing the points that represent the roots of the equation $z^4 + 3z^2 - 4 = 0$. [2]

4 The complex number z is given by $-20 + 21\mathrm{i}$. Showing all your working,

(i) find the value of $|z|$, [2]

(ii) calculate the value of $\arg z$ correct to 3 significant figures, [2]

(iii) express $\dfrac{1}{z}$ in the form $x + \mathrm{i}y$, where x and y are real numbers. [2]

5 **(i)** Verify that $z = -1$ is a root of the equation $z^3 + 5z^2 + 9z + 5 = 0$. [1]

(ii) Find the two complex roots of the equation $z^3 + 5z^2 + 9z + 5 = 0$. [4]

(iii) Show all three roots on an Argand diagram. [1]

6 The roots of the equation $z^2 - 6z + 10 = 0$ are z_1 and z_2, where $z_1 = 3 + \mathrm{i}$.

(i) Write down the value of z_2. [1]

(ii) Show z_1 and z_2 on an Argand diagram. [2]

(iii) Show that $z_1^2 = 8 + 6\mathrm{i}$. [2]

7 **(a)** The complex number z is such that $|z| = 2$ and $\arg z = -\frac{2}{3}\pi$. Find the exact value of the real part of z and of the imaginary part of z. [2]

(b) The complex numbers u and v are such that

$$u = 1 + \mathrm{i}a \quad \text{and} \quad v = b - \mathrm{i},$$

where a and b are real and $a < b$. Given that $uv = 7 + 9\mathrm{i}$, find the values of a and b. [7]

8 **(a)** Solve the equation

$$(2 + \mathrm{i})z = (4 + \mathrm{i}n).$$

Give your answer in the form $a + \mathrm{i}b$, expressing a and b in terms of the real constant n. [4]

(b) The roots of the equation $z^2 + 8z + 25 = 0$ are denoted by z_1 and z_2.

(i) Find z_1 and z_2 and show these roots on an Argand diagram. [3]

(ii) Find the modulus and argument in radians of each of $(z_1 + 1)$ and $(z_2 + 1)$. [3]

9 It is given that

$$y = \frac{1}{x + \mathrm{i}} + \frac{1}{x - \mathrm{i}},$$

where x and y are real and positive, and $\mathrm{i}^2 = -1$.

(i) Show that

$$x = \frac{1 \pm \sqrt{1 - y^2}}{y} \quad \text{and} \quad y \leqslant 1.$$ [4]

(ii) Deduce that

$$xy < 2.$$ [2]

(iii) Indicate the region in the x-y plane defined by

$$y \leqslant 1 \quad \text{and} \quad xy < 2.$$ [3]