



# Cambridge O Level

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**ADDITIONAL MATHEMATICS**

**4037/12**

Paper 1

**October/November 2020**

MARK SCHEME

Maximum Mark: 80

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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This document consists of **9** printed pages.

**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

### MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

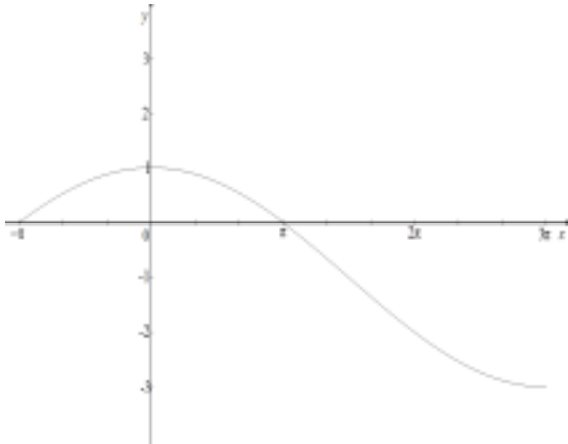
#### Types of mark

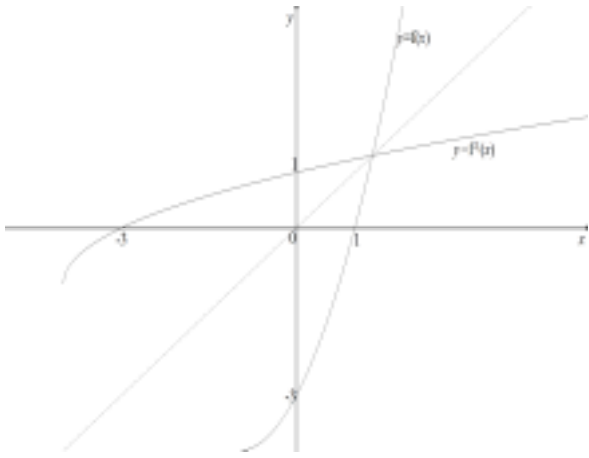
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

#### Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$2x^2 - (k+4)x + (k+4) (=0)$ $2x^2 + (-k-4)x + (k+4) (=0)$	<b>B1</b>	
	Discriminant: $(k+4)^2 - (4 \times 2 \times (k+4))$	<b>M1</b>	Use of discriminant to obtain 2 critical values using <i>their</i> 3 term quadratic
	$\pm 4$	<b>A1</b>	For critical values
	$k < -4$ $k > 4$	<b>A1</b>	
2(a)	$y = -\frac{1}{2}(x+5)(x+1)(x-2)$	<b>3</b>	<b>B1</b> for negative soi <b>B1</b> for $\frac{1}{2}$ soi <b>B1</b> for $(x+5)(x+1)(x-2)$ or $x^3 + 4x^2 - 7x - 10$
2(b)	$-5 < x < -1$	<b>B1</b>	
	$x > 2$	<b>B1</b>	
3(a)	2	<b>B1</b>	
3(b)	$6\pi$ or $1080^\circ$	<b>B1</b>	
3(c)		<b>3</b>	<b>B1</b> for passing through $(-\pi, 0)$ and $(3\pi, -3)$ – must be a curve <b>B1</b> for correct shape with max on $y$ -axis and a min at $x = 3\pi$ <b>B1</b> for passing through $(0, 1)$ and $(\pi, 0)$ only on the positive $x$ -axis
4(a)	$a + 6d = 158$ $a + 9d = 149$	<b>B1</b>	For both equations, may be implied by a correct $a$ and $d$
	$d = -3,$	<b>B1</b>	
	$a = 176$	<b>B1</b>	

Question	Answer	Marks	Guidance
4(b)	$\frac{n}{2}(352 + (n-1)(-3)) \quad (< 0)$	<b>M1</b>	For correct attempt at sum formula with <i>their a</i> and <i>their d</i> ,
	$\frac{355}{3}$ or 118.3 oe	<b>A1</b>	
	119	<b>A1</b>	
5	$x^5 + 10x^3 + 40x + \dots$	<b>3</b>	<b>M1</b> for attempt to expand $\left(x + \frac{2}{x}\right)^5$ , with at least 2 correct terms <b>A1</b> for $10x^3$ <b>A1</b> for $40x$
	Term in $x^2$ : $(1 \times 40) - (3 \times 10)$	<b>M1</b>	For $(1 \times \textit{their } 40) \pm (3 \times \textit{their } 10)$
	10	<b>A1</b>	
6(a)	It is a one-one function because of the given restricted domain or because $x \geq -1$	<b>B1</b>	
6(b)		<b>4</b>	<b>B1</b> for $y = f(x)$ for $x > -1$ only <b>B1</b> for 1 on x-axis and $-3$ on y-axis for $y = f(x)$ <b>B1</b> for $y = f^{-1}(x)$ as a reflection of $y = f(x)$ in the line $y = x$ , maybe implied by intercepts with axes <b>B1</b> for 1 on y-axis and $-3$ on x-axis for $y = f^{-1}(x)$

Question	Answer	Marks	Guidance
7(a)	$\frac{dy}{dx} = \frac{(2x+1)\frac{6x}{3x^2-5} - 2\ln(3x^2-5)}{(2x+1)^2}$ or $\frac{dy}{dx} = (2x+1)^{-1} \frac{6x}{3x^2-5} - 2(2x+1)^{-2} \ln(3x^2-5)$	3	<b>B1</b> for $\frac{6x}{3x^2-5}$ <b>M1</b> for attempt at a quotient or equivalent product <b>A1</b> for all terms other than $\frac{6x}{3x^2-5}$ correct
	When $x = \sqrt{2}$ , $y = 0$	<b>B1</b>	May be implied
	When $x = \sqrt{2}$ , $\frac{dy}{dx} = \frac{6\sqrt{2}}{2\sqrt{2}+1}$ or $\frac{24-6\sqrt{2}}{7}$ or 2.22 oe Normal: $y = -\frac{(2\sqrt{2}+1)}{6\sqrt{2}}(x-\sqrt{2})$ oe or $y = -\frac{7}{24-6\sqrt{2}}(x-\sqrt{2})$ oe or $y = -\frac{1}{2.22}(x-\sqrt{2})$ oe or $y = -\frac{4+\sqrt{2}}{12}(x-\sqrt{2})$ oe or $y = -\frac{9+4\sqrt{2}}{24+6\sqrt{2}}(x-\sqrt{2})$ oe $y = -0.451x + 0.638$	2	<b>M1</b> for attempt at normal using <i>their</i> $y$ and <i>their</i> perp gradient <b>A1</b> Allow equivalent surd forms
7(b)	$\left(\frac{6\sqrt{2}}{2\sqrt{2}+1}\right)h$ or $\frac{24-6\sqrt{2}}{7}h$ or other equivalent surd forms, or 2.22h	<b>B1</b>	<b>FT</b> on <i>their</i> $\frac{dy}{dx}$ from (a)
8(a)	${}^{12}C_3 \times {}^9C_4 = 220 \times 126$ or ${}^{12}C_5 \times {}^7C_4 = 792 \times 35$ or ${}^{12}C_4 \times {}^8C_5 = 495 \times 56$ or other equivalents 27720	3	<b>B1</b> for one correct combination in a product of 2 or 3 combinations Must be numeric <b>B1</b> for a second appropriate combination in the same product Must be numeric
8(b)(i)	120	<b>B1</b>	
8(b)(ii)	48	<b>B1</b>	

Question	Answer	Marks	Guidance								
8(b)(iii)	Starts with 7 or 9      24	<b>B1</b>	May be implied by 12 and 12								
	Starts with 8      18	<b>B1</b>									
	42	<b>B1</b>									
	<b>Alternative</b> Ends with 3      18	<b>(B1)</b>									
	Ends with 7 or 9      24	<b>(B1)</b>	May be implied by 12 and 12								
	42	<b>(B1)</b>									
9(a)	$\frac{dy}{dx} = (2x-1) \times \frac{1}{2} \times 4(4x+3)^{-\frac{1}{2}} + 2(4x+3)^{\frac{1}{2}}$	<b>3</b>	<b>B1</b> for $\frac{1}{2} \times 4(4x+3)^{-\frac{1}{2}}$ oe <b>M1</b> for a correct attempt at a product <b>A1</b> for all other terms correct								
	$\frac{dy}{dx} = 2(4x+3)^{-\frac{1}{2}}(2x-1+4x+3)$ or equivalent	<b>M1</b>	For attempt to simplify to the given form								
	$\frac{dy}{dx} = \frac{4(3x+1)}{(4x+3)^{\frac{1}{2}}}$	<b>A1</b>									
9(b)	$-\frac{1}{3}$	<b>B1</b>	<b>FT</b> on <i>their</i> $3x+1=0$								
9(c)	For a complete method using 2 <sup>nd</sup> derivative, or gradient or y values either side or one side of <i>their</i> stationary point e.g.	<b>M1</b>	Must be using values of $x > -\frac{3}{4}$								
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>&lt; -\frac{1}{3}</math></td> <td style="padding: 5px;"><math>-\frac{1}{3}</math></td> <td style="padding: 5px;"><math>&gt; -\frac{1}{3}</math></td> </tr> <tr> <td style="padding: 5px;"><math>\frac{dy}{dx}</math></td> <td style="padding: 5px;">-</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">+</td> </tr> </table>			$x$	$< -\frac{1}{3}$	$-\frac{1}{3}$	$> -\frac{1}{3}$	$\frac{dy}{dx}$	-	0	+
	$x$			$< -\frac{1}{3}$	$-\frac{1}{3}$	$> -\frac{1}{3}$					
	$\frac{dy}{dx}$	-	0	+							
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>&lt; -\frac{1}{3}</math></td> <td style="padding: 5px;"><math>-\frac{1}{3}</math></td> <td style="padding: 5px;"><math>&gt; -\frac{1}{3}</math></td> </tr> <tr> <td style="padding: 5px;"><math>y</math></td> <td style="padding: 5px;"><math>&lt; -2.15</math></td> <td style="padding: 5px;"><math>-2.15</math></td> <td style="padding: 5px;"><math>&gt; -2.15</math></td> </tr> </table>	$x$	$< -\frac{1}{3}$	$-\frac{1}{3}$	$> -\frac{1}{3}$	$y$	$< -2.15$	$-2.15$	$> -2.15$			
$x$	$< -\frac{1}{3}$	$-\frac{1}{3}$	$> -\frac{1}{3}$								
$y$	$< -2.15$	$-2.15$	$> -2.15$								
Minimum	<b>A1</b>	Must be from correct work									

Question	Answer	Marks	Guidance
10(a)	$p(2): 48 + 4a + 2b + 2 = 0$ $2a + b + 25 = 0$	<b>B1</b>	For $2a + b + 25 = 0$ or multiple
	$p(1) = -2p(0)$ $a + b + 12 = 0$	<b>B1</b>	For $a + b + 12 = 0$
	$a = -13, \quad b = 1$	<b>2</b>	<b>M1</b> for attempt to solve <i>their</i> equations in $a$ and $b$ leading to 2 values <b>A1</b> for both
10(b)(i)	$p\left(\frac{1}{2}\right) = \frac{6}{8} - \frac{13}{4} + \frac{1}{2} + 2$	<b>M1</b>	For attempt to find $p\left(\frac{1}{2}\right)$ using <i>their</i> $a$ and $b$
	0	<b>A1</b>	
10(b)(ii)	$(x - 2)(2x - 1)(3x + 1)$	<b>2</b>	<b>M1</b> for realising that 2 factors are known and 3 <sup>rd</sup> factor can be got by observation or algebraic long division, or for making use of $x - 2$ or $2x - 1$ in order to obtain a quadratic factor <b>A1</b> Must see all factors together
11(a)	$\angle BOC = 1.5 \text{ rad}$	<b>B1</b>	
	$\sin 0.75 = \frac{BC/2}{r}$	<b>M1</b>	For a complete attempt to find $BC$ – must be using a right-angled triangle to get required result – <b>Given answer</b>
	$BC = 2r \sin 0.75$	<b>A1</b>	
	Perimeter = $2r + 2r \sin 0.75 + 4r + 1.5r$	<b>M1</b>	<b>Dep</b> on first M mark for attempt at perimeter
	$r(7.5 + 2 \sin 0.75)$	<b>A1</b>	<b>Given answer</b>
11(b)	Area = $(2r + 2r \sin 0.75)r - \frac{1}{2}r^2(1.5 - \sin 1.5)$ Area = $3.36r^2 - 0.75r^2 + 0.4987r^2$	<b>3</b>	<b>M1</b> for a correct plan <b>M1</b> for $(2r + 2r \sin 0.75)r$ , using <i>their</i> $2r \sin 0.75$ <b>B1</b> for segment $\frac{1}{2}r^2(1.5 - \sin 1.5) = 0.251r^2$
	Area = $3.11r^2$	<b>A1</b>	



Question	Answer	Marks	Guidance
12(a)(i)	Area under graph: $\frac{1}{2}(60+40)\times 30 + \frac{1}{2}(30+V)\times 30 \quad (=2775)$ or $\frac{1}{2}(20\times 30) + (40+30) + \frac{1}{2}(30+V)\times 30$	<b>2</b>	<b>M1</b> for attempt to find the area under the graph <b>Dep M1</b> on previous M mark for attempt to equate to 2775 and simplify in order to find $V$ or $V - 30$
	55	<b>A1</b>	
12(a)(ii)	0	<b>B1</b>	
12(b)(i)	$v = 3\sin 2t \quad (+c)$	<b>M1</b>	Must have $\pm 3\sin 2t$
	$10 = c$	<b>M1</b>	<b>Dep</b> for attempt to find $+c$ ,
	$v = 3\sin 2t + 10$	<b>A1</b>	
12(b)(ii)	$s = -\frac{3}{2}\cos 2t + 10t + d$	<b>M1</b>	For attempt to integrate <i>their</i> $v$ , must have $\pm \frac{3}{2}\cos 2t$
	$d = \frac{3}{2}$	<b>M1</b>	<b>Dep</b> on previous M mark for attempt to find $d$ .
	$s = -\frac{3}{2}\cos 2t + 10t + \frac{3}{2}$	<b>A1</b>	