Cambridge Pre-U

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Question	Answer	Marks	Guidance
1	$2x - 3 = x - 1 \Rightarrow x = 2$	**B1**	Obtain $x = 2$
	$2x - 3 = 1 - x$	**M1**	Solve equation "other way round"
	$x = \frac{4}{3}$	**A1**	Obtain $x = \frac{4}{3}$
	Alternative method for question 1		
	$4x^2 - 12x + 9 = x^2 - 2x + 1$	**M1**	Squaring both sides and creating a quadratic eqn.
	$0 = 3x^2 - 10x + 8 = (3x - 4)(x - 2)$	**M1**	Method for solving a quadratic eqn.
	$x = \frac{4}{3}, 2$	**A1**	Obtain both correct answers
	Alternative method for question 1		
	Attempt at two modulus graphs	**M1**	
	$x = \frac{4}{3}$	**A1**	First answer correctly obtained
	$x = 2$	**A1**	Second answer correctly obtained

Question	Answer	Marks	Guidance
2(a)	$m = \dfrac{3-7}{9-1} = -\dfrac{1}{2}$	**B1**	Correct gradient of PQ
	mid-point = (5, 5)	**B1**	Correct mid-point of PQ
	$y - 5 = 2(x - 5)$	**M1**	Attempt equation of line through their mid-point, with gradient $\dfrac{-1}{\text{their } m}$
	$y = 2x - 5$	**A1**	Obtain correct equation
2(b)	$(\frac{5}{2}, 0)$ and $(0, -5)$	**M1**	Attempt intercepts on x- and y-axes *and* triangle area formula
	area = $\frac{1}{2} \times \frac{5}{2} \times 5 = \frac{25}{4}$	**A1ft**	Obtain correct area of triangle, a.e.f. Allow clearly demonstrated **FT** answers

Question	Answer	Marks	Guidance
3(a)	$-x^2 + 5x - 4$	**B1**	Correct expansion
	$\frac{9}{4} - (x - \frac{5}{2})^2$	**B1**	Correct $b = \frac{5}{2}$
		B1	Correct $a = \frac{9}{4}$ (allow incorrect sign for b)

Question	Answer	Marks	Guidance
3(b)	$\left(\frac{5}{2}, 3\right)$	**B1ft**	FT their part **(a)** answer: x-coord. = b
		B1ft	FT their part **(a)** answer: y-coord. = $a + \frac{3}{4}$
	Alternative method for question 3(b)		
	Use of calculus to find a turning point	**B1 B1**	Finding correct TP; proving this is a max.

Question	Answer	Marks	Guidance
4(a)	$\overrightarrow{AB} = -5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	**B1**	Correct AB or BA seen
	$10\mathbf{i} + a\mathbf{j} + b\mathbf{k} = -2(-5\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**	Attempt to use a scalar multiple of their direction vector
	$a = -4$, $b = -2$	**A1**	Obtain correct values for a and b
4(b)	$-10 + 2 + c = 0$	**M1**	Setting a scalar product = 0 using either their AB, or their ($10\mathbf{i} + a\mathbf{j} + b\mathbf{k}$) from part **(a)**
	$c = 8$	**A1**	Obtain $c = 8$

Question	Answer	Marks	Guidance
5	$\frac{6}{x} = 7 - x \Rightarrow x^2 - 7x + 6 = 0 \Rightarrow (x-1)(x-6) = 0$	**M1**	Attempt to find points of intersection
	$x = 1, x = 6$	**A1**	Obtain both correct x-coordinates
	$\int \frac{6}{x} \mathrm{d}x = 6\ln\lvert x\rvert$	**M1**	Integration to obtain $k \ln(x)$, or equivalent (modulus signs not required)
	$\left[6\ln x\right]_1^6 = 6\ln 6$	**M1**	Attempted use of their x limits (must be positive) – correct order and subtraction (condone ln1 unseen)
	$\frac{1}{2} \times 5 \times (1+6) = \frac{35}{2}$ or $\int (7-x)\,\mathrm{d}x = [7x - \frac{1}{2}x^2]$	**M1**	Attempt trapezium area
	Area = $\frac{35}{2} - 6\ln 6$	**A1**	Obtain $\frac{35}{2} - 6\ln 6$, or any exact equivalent form

Question	Answer	Marks	Guidance
6	$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha = 7\cos\theta - 4\sin\theta$	**M1**	Attempt to use $R\cos(\theta+\alpha)$ Allow other identities, as α not defined as acute
	$R = \sqrt{65}$ ($\approx 8.062\ldots$)	**A1**	Obtain correct value for R (allow 8.1 or better)
	$\tan\alpha = \frac{4}{7}$	**M1**	Attempt valid method to find α
	$\alpha = 29.7°$	**A1**	Obtain $\alpha = 29.7°$ or $\tan^{-1}\frac{4}{7}$
	$\sqrt{65}\cos\left(\theta + 29.7°\right) = 3$ or $\Rightarrow \theta + 29.7° = \cos^{-1}\frac{3}{\sqrt{65}}$	**M1**	Attempt correct order of operations to find at least one root (68.155…°, 291.845…°)
	$\theta = 38.4°$ (a.w.r.t.)	**A1**	Obtain 1st solution, correct to at least 1d.p.
	$\theta = 262.1°$ (a.w.r.t.)	**A1**	Obtain 2nd solution, correct to at least 1d.p. but withhold 2nd A mark if any extra solutions offered
	Alternative method for question 6		
	$7c = 3 + 4s \Rightarrow 49c^2 = 9 + 24s + 16s^2$	**M1**	Isolating one trig. term and squaring
	$\Rightarrow 49 - 49s^2 = 9 + 24s + 16s^2$	**M1**	Use of $c^2 + s^2 = 1$ to obtain an eqn. in s or c only
	$\Rightarrow 0 = 65s^2 + 24s - 40$	**A1**	Correct quadratic eqn. in s or c
	$\sin\theta = 0.6213$ or $\sin\theta = -0.9905$	**M1 A1**	Solving quadratic eqn. in s or c; 2 correct values
	giving $\theta = 38.4°, 141.6°$ or $\theta = 262.1°, 277.9°$	**A1**	Obtain 1st solution, correct to at least 1d.p.
	Checking to leave $\theta = 38.4°, 262.1°$	**A1**	Obtain 2nd solution, correct to at least 1d.p. but withhold 2nd A mark if extra solutions are offered

Question	Answer	Marks	Guidance
7	$\dfrac{a}{1-r}=8$	**B1**	Correct sum to infinity statement (all terms)
	$\dfrac{ar}{1-r^2}=2$	**B1**	Correct S_∞ statement for even-numbered terms **or** $\dfrac{a}{1-r^2}=6$ (i.e. for S_∞ of odd-numbered terms)
	$\dfrac{r}{1+r}=\frac{1}{4}$	**M1**	Attempt to eliminate one variable
		A1	Correct
	$4r=1+r$	**M1**	Formulate linear (or quadratic) eqn. in r and solve it
	$r=\frac{1}{3}$	**A1**	Obtain correct r (one value only)
	$a=\frac{16}{3}$	**A1**	Obtain correct a (don't penalise 2nd soln. twice)
	Alternative method for question 7		
	$\dfrac{a}{1-r}=8$	**B1**	Correct sum to infinity statement (all terms)
	$\dfrac{ar}{1-r^2}=2$	**B1**	Correct S_∞ statement for even-numbered terms **or** $\dfrac{a}{1-r^2}=6$ (i.e. for S_∞ of odd-numbered terms)
	$(ar=)\ 8r-8r^2=2-2r^2$	**M1**	Attempt to eliminate one variable
	$\Rightarrow 6r^2-8r+2=0$	**A1**	Obtain correct three term quadratic in one variable
	$\Rightarrow (3r-1)(r-1)=0$	**M1**	Attempt to solve quadratic eqn.
	$r=\frac{1}{3}$	**A1**	Obtain correct r (A0 if $r=1$ also given)

Question	Answer	Marks	Guidance
7	$a = \frac{16}{3}$	**A1**	Obtain correct *a* (don't penalise 2nd soln. twice)

Question	Answer	Marks	Guidance
8(a)	$\frac{z^2}{zz^*} = \frac{1}{5}(3+4\text{i}) \Rightarrow x^2 - y^2 + 2xy\text{i} = 3 + 4\text{i}$	**M1**	Using $zz^* = 5$ and squaring $(x + \text{i}y)$
		M1	Equating Re and/or Im parts
	(**Re**) $x^2 - y^2 = 3$ (**Im**) $xy = 2$ and $x^2 + y^2 = 5$ from $zz^* = 5$	**A1**	Two correct eqns. in x, y
	e.g. $x^2 + y^2 = 5$ and $x^2 - y^2 = 3 \Rightarrow 2x^2 = 8$ **or** $x^2 - \left(\frac{2}{x}\right)^2 = 3 \Rightarrow x^4 - 3x^2 - 4 = 0$	**M1**	Using two of these three eqns. “simultaneously” to eliminate one variable
	$(x^2 - 4)(x^2 + 1) = 0 \Rightarrow x = \pm 2$	**M1**	Solving to find x or y (at least one value)
	$z = 2 + \text{i}$, $z = -2 - \text{i}$	**A1**	Obtain both correct complex numbers
	Alternative method for question 8(a)		
	Multiplying by $z^* = x - \text{i}y \Rightarrow x + \text{i}y = (\frac{3}{5} + \frac{4}{5}\text{i})(x - \text{i}y)$	**M1**	Multiplying up by $z^* = x - \text{i}y$
	$\Rightarrow x + \text{i}y = (\frac{3}{5}x + \frac{4}{5}y) + \text{i}(\frac{4}{5}x - \frac{3}{5}y)$	**M1**	Equating Re and/or Im parts
	Then $x = \frac{3}{5}x + \frac{4}{5}y$ and $y = \frac{4}{5}x - \frac{3}{5}y \Rightarrow x = 2y$ $zz^* = 5 \Rightarrow x^2 + y^2 = 5$	**A1**	Obtaining two correct eqns. in x, y
		M1	Method for eliminating one variable
	e.g. Setting $z = 2y + \text{i}y$ and using $zz^* = \lvert z \rvert^2 \Rightarrow 5y^2 = 5$	**M1**	Solving to find x or y (at least one value)
	so that $y = \pm 1$ and $z = \pm(2 + \text{i})$	**A1**	Obtain both correct complex numbers

Question	Answer	Marks	Guidance
8(b)	Line with –ve gradient cutting positive x- and y-axes	**M1**	**Or** Attempt to calculate cartesian eqn.
	Correct line, $y = -2x + 3$, shown on Argand diagram	**A1**	Possibly implied by line through (0, 3) and ($\frac{3}{2}$, 0) **or** geometrically as line perpr. to line segment joining 4 and –2i thro' its midpoint

Question	Answer	Marks	Guidance
9(a)	$\cos\alpha = \dfrac{(p-q)^2 + p^2 - (p+q)^2}{2p(p-q)}$	**M1**	Use of cosine rule in correct arrangement
	$\cos\alpha = \dfrac{(p^2 - 2pq + q^2) + p^2 - (p^2 + 2pq + q^2)}{2p(p-q)}$	**M1**	Expand brackets and attempt to simplify numerator
	$\cos\alpha = \dfrac{p^2 - 4pq}{2p(p-q)}$	**A1**	Numerator and denominator correct (unsimplified) o.e.
	$= \dfrac{p(p-4q)}{2p(p-q)} \Rightarrow \cos\alpha = \dfrac{p-4q}{2(p-q)}$ **AG**	**A1**	Given answer obtained from suitable working

Question	Answer	Marks	Guidance
9(b)	$\cos\alpha = \frac{1}{4} \Rightarrow \sin\alpha = \frac{\sqrt{15}}{4}$	B1ft	Correct sine obtained from cosine (after substituting $p = 7$, $q = 1$). Accept $\alpha = 75.5°$ (1.318 rads) here.
	Use of area $\Delta =$ "$\frac{1}{2}ab\sin\alpha$"	M1	With a, b any two of side-lengths 6, 7, 8
	$= \frac{21}{4}\sqrt{15}$	A1	Correct exact answer, any equivalent form
9(c)	$-\frac{\sqrt{3}}{2} \times 2(p-q) = p - 4q$	B1	Use of cos 150° = $-\frac{\sqrt{3}}{2}$ in expression from **(a)**
	$-\sqrt{3}p + \sqrt{3}q = p - 4q$ $p + \sqrt{3}p = 4q + \sqrt{3}q$ or $p\left(1+\sqrt{3}\right) = q\left(4+\sqrt{3}\right)$	M1	Expand and collect like terms
	$p = \left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)q$	A1	Obtain correct final answer, a.e.f. including rationalised denominator $\left(p = \frac{-1+3\sqrt{3}}{2}q\right)$

Question	Answer	Marks	Guidance
10(a)(i)	$\dfrac{dy}{dx}=\dfrac{(-2x)(1+x^2)-(1-x^2)(2x)}{(1+x^2)^2}$	**M1**	Attempt quotient rule
		A1	Correct unsimplified expression
	$\dfrac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}=\dfrac{-4x}{(1+x^2)^2}$ **AG**	**A1**	Obtain given answer; detail required
	Alternative method for question 10(a)(i)		
	$y=\dfrac{2}{1+x^2}-1=2(1+x^2)^{-1}-1$	**B1**	
	$\dfrac{dy}{dx}=-2(1+x^2)^{-2}.\,2x=$ **AG**	**M1**	Differentiation using the chain rule
		A1	Answer in given form supported
10(a)(ii)	$\dfrac{d}{dx}\left(1+x^2\right)^2=4x\left(1+x^2\right)$	**B1**	Correct derivative of $(1+x^2)^2$ seen, any form
	$\dfrac{d^2y}{dx^2}=\dfrac{-4(1+x^2)^2-(-4x)(4x(1+x^2))}{(1+x^2)^4}$	**M1**	Attempted use of the quotient rule
		A1	Correct unsimplified second derivative
	$\dfrac{d^2y}{dx^2}=\dfrac{12x^4+8x^2-4}{(1+x^2)^4}$ or $\dfrac{(1+x^2)(12x^2-4)}{(1+x^2)^4}$ or $\dfrac{4(3x^2-1)}{(1+x^2)^3}$	**A1**	Correct simplified second derivative (factor of 4 may be inside bracket in final form)
	Alternative method for question 10(a)(ii)		
	$\dfrac{d}{dx}\left(1+x^2\right)^2=4x\left(1+x^2\right)$	**B1**	Correct derivative of $(1+x^2)^2$ seen, any form

Question	Answer	Marks	Guidance
10(a)(ii)	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{-4(1+x^2)^2-(-4x)(4x(1+x^2))}{(1+x^2)^4}$	**M1**	Attempted use of the quotient rule
		A1	Correct unsimplified second derivative
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-4}{(1+x^2)^2}+\dfrac{16x^2}{(1+x^2)^3}$ **or** $\dfrac{12}{(1+x^2)^2}-\dfrac{16}{(1+x^2)^3}$	**A1**	Alt. answer forms may appear

Question	Answer	Marks	Guidance
10(b)	$y = 0 \Rightarrow x = \pm 1$	**B1**	Identify coordinates of A and B
	$y = -1(x-1)$ $\quad y = 1(x+1)$ $y = -x+1$ $\quad y = x+1$	**M1**	Attempt at equations of both tangents
	intersect at (0, 1), hence C is on the y-axis	**A1**	Correct conclusion, from correct equations
	Alternative method for question 10(b)		
	Symmetry argument: curve is that of an even function	**B1**	No justification required
	so tgt. at A is mirror image of tgt. at B	**M1**	
	and these must meet on the y-axis	**A1**	
10(c)	In curve's eqn. $x = 0 \Rightarrow y = 1$ hence C is a point on the curve	**B1**	
	$x = 0 \Rightarrow \frac{dy}{dx} = 0$ hence C is a stationary point	**B1**	
	$x = 0 \Rightarrow \frac{d^2y}{dx^2} = -4 < 0$ hence C is a maximum point **OR** Sign of $\frac{dy}{dx}$ evaluated either side of $x = 0$ **OR** graphical or symmetry argument	**B1ft**	Second derivative test to show that C is a max. (**FT** their y'' provided it gives a negative value) Numerical evidence must be seen Or use of y-value either side of $x = 0$

Question	Answer	Marks	Guidance
11	$\mathrm{d}u = \mathrm{e}^x\mathrm{d}x$	**B1**	Correct statement linking du and dx
	$\int \frac{10\mathrm{e}^x}{\mathrm{e}^{2x} - 3\mathrm{e}^x - 4}\mathrm{d}x = \int \frac{10}{u^2 - 3u - 4}\mathrm{d}u$	**M1**	Attempt to rewrite in terms of u
		A1	Fully correct integrand, including du implied (all remaining marks possible following no du)
	$\frac{10}{u^2 - 3u - 4} = \frac{A}{u-4} + \frac{B}{u+1} \Rightarrow A(u+1) + B(u-4) = 10$	**M1**	Attempt partial fractions
	$A = 2$, $B = –2$	**A1**	Obtain correct partial fractions
	$\int \left(\frac{2}{u-4} - \frac{2}{u+1}\right)\mathrm{d}u = 2\ln\lvert u-4\rvert - 2\ln\lvert u+1\rvert$	**A1ft**	Correct integration of their partial fractions Modulus brackets not required
	$\left(2\ln\left(\mathrm{e}^k - 4\right) - 2\ln\left(\mathrm{e}^k + 1\right)\right) - \left(2\ln 1 - 2\ln 6\right)$	**M1**	Attempt use of limits, using k and ln5 in an integral involving x or e^k and 5 in an integral involving u
	$2\ln\left(\frac{6\left(\mathrm{e}^k - 4\right)}{\mathrm{e}^k + 1}\right) = 2\ln 5$	**M1**	Equate to ln25, and use log laws correctly to remove logs
	$\Rightarrow \frac{6\mathrm{e}^k - 24}{\mathrm{e}^k + 1} = 5$	**A1**	Obtain correct equation not involving logs
	$6\mathrm{e}^k - 24 = 5\mathrm{e}^k + 5 \Rightarrow \mathrm{e}^k = 29 \Rightarrow k = \ln 29$	**A1**	Obtain k = ln29