Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5 [Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type B:
Each question is followed by its answer scheme

Chapter 4

Discrete random variables

193. 9709_m22_qp_52 Q: 1

A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

(b) Given that $E(X) = 0.25$, find the value of $\mathrm{Var}(X)$. [2]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">(a)</td><td rowspan="3"><table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td rowspan="2">P(X)</td><td>$\frac{1}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{2}{16}$</td></tr>
<tr><td>0.0625</td><td>0.1875</td><td>0.3125</td><td>0.3125</td><td>0.125</td></tr>
</table></td><td>B1</td><td>Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.</td></tr>
<tr><td>B1</td><td>3 correct probabilities linked with correct outcomes, may not be in table.</td></tr>
<tr><td>B1</td><td>2 further correct probabilities linked with correct outcomes, may not be in table
No repeated X values.

SC if less than 3 correct probabilities seen, award
SCB1 Sum of <i>their</i> probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1±0·01 or better).</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td>$\left[\frac{1}{16}\times-2^2+\frac{3}{16}\times-1^2\left(+\frac{5}{16}\times0^2\right)+\frac{5}{16}\times1^2+\frac{2}{16}\times2^2-\left(\frac{1}{4}\right)^2\right]$
$\frac{1\times4+3\times1+5\times0+5\times1+2\times4}{16}-0.25^2$</td><td>M1</td><td>Appropriate variance formula using $(\mathrm{E}(X))^2$ value, accept unsimplified. FT <i>their</i> table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$.
Condone 1 error providing all probabilities <1 and 0.25^2 used</td></tr>
<tr><td>$\left[=\frac{5}{4}-\frac{1}{16}=\right]\frac{19}{16}$, 1.1875</td><td>A1</td><td>Condone 1.188 or 1.19 WWW</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

194. 9709_m22_qp_52 Q: 2

In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

....

....

....

....

....

....

....

....

....

....

(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	[P(>2) = 1 – P(0,1,2) =] $1 - ({}^7C_0\ 0.18^0\, 0.82^7 + {}^7C_1\ 0.18^1\, 0.82^6 + {}^7C_2\ 0.18^2\, 0.82^5)$	**M1**	One term ${}^7C_x\ p^x(1-p)^{7-x}$, $0<p<1, 0<x<7$
	= 1 – (0.249285 + 0.383048 + 0.252251) = 1 – 0.88458	**A1**	Correct unsimplified expression or better Condone omission of brackets if recovered
	0.115	**B1**	WWW. $0{\cdot}115 \leqslant p < 0{\cdot}1155$ not from wrong working
		3	
(b)	[P(at least 1 day of rain) = 1 – P(0) = $1-(0.82)^7$ =] 0.7507	**B1**	AWRT 0.751 seen
	[P(exactly 2 periods) =] $0.7507^2 \times (1-0.7507) \times 3$	**M1**	FT *their* $1-p^7$ or *their* 0.7507 if identified, not 0.18, 0.82 Accept $\times{}^3C_r$, r=1,2 or $\times{}^3P_1$ for ×3 Condone ×2
	0.421	**A1**	Accept $0.421 \leqslant p \leqslant 0.4215$ **SC B1** if 0/3 scored for final answer only $0.421 \leqslant p \leqslant 0.4215$
		3	

195. 9709_m22_qp_52 Q: 6

A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

....

....

....

....

....

....

....

....

....

....

....

(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	[Probability of lemon = $\frac{3}{15}=\frac{1}{5}$] $\left[\left(\frac{4}{5}\right)^6\times\frac{1}{5}=\right]\frac{4096}{78125}$, 0.0524	**B1**	0.0524288 rounded to more than 3SF if final answer
		1	
(b)	$\left(1-\frac{1}{5}\right)^6$	**M1**	or $\left(\frac{4}{5}\right)^6$. FT *their* $\frac{1}{5}$ or correct. From final answer Condone $\left(\frac{4}{5}\right)^5$ or $\left(\frac{1}{5}\right)\times\left(\frac{4}{5}\right)^5+\left(\frac{4}{5}\right)^6$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
	Alternative method for question 6(b)		
	[1 – P(1,2,3,4,5,[6]) =] $1-\left(\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}+\left(\frac{4}{5}\right)^5\times\frac{1}{5}\right)$	**M1**	From final answer Condone omission of $\left(\frac{4}{5}\right)^5\times\frac{1}{5}$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
		2	
(c)	$\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}$	**M1**	$\frac{a}{15}\times\frac{a-1}{14}\times\frac{a-2}{13}$, no additional terms
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{3}{15}\times\frac{2}{14}\times\frac{1}{13}+3\times\frac{3}{15}\times\frac{2}{14}\times\frac{7}{13}+3\times\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}+\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$	**M1**	[3Ls + 2Ls1S + 1L2Ss + 3Ss] Condone one numerator error. Condone no multiplications seen if tree diagram complete with probabilities on each branch, scenarios listed and attempt at evaluation
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$1-\left(\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}+3\times\frac{5}{15}\times\frac{4}{14}\times\frac{10}{13}+3\times\frac{5}{15}\times\frac{10}{14}\times\frac{9}{13}\right)$	**M1**	1 – P(3,2,1 oranges) Condone one numerator error.
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{{}^{10}C_3}{{}^{15}c_3}$	**M1**	
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
(d)	$\frac{7}{15}\times\frac{5}{14}\times\frac{3}{13}\times 3!$	M1	All probabilities of the form: $\frac{7}{a}\times\frac{5}{b}\times\frac{3}{c}$, $13 \leqslant a,b,c \leqslant \underline{15}$
		M1	$\frac{e}{f}\times\frac{g}{h}\times\frac{i}{j}\times 3!$ e,f,g,h,i,j positive integers forming probabilities or 6 identical probability calculations or values added, no additional terms
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
	Alternative method for question 6(d)		
	$\frac{{}^{3}C_{1}\times{}^{5}C_{1}\times{}^{7}C_{1}}{{}^{15}C_{3}}$	M1	$\frac{{}^{3}C_{1}\times{}^{5}C_{1}\times{}^{7}C_{1}}{k}$, k integer > 1 Condone use of permutations
		M1	$\frac{{}^{3}C_{a}\times{}^{5}C_{b}\times{}^{7}C_{c}}{{}^{15}C_{3}}$, $0<a<3$, $0<b<5$, $0<c<7$, Condone use of permutations
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
		3	

Question	Answer	Marks	Guidance
(e)	$\frac{\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}+\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3}{their(c)}$ $\left[=\frac{14}{65}\div\frac{24}{91}\right]$	B1	$\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3$ seen (SSL, SLS, LSS) **SC B1** $\frac{3}{65}\times 3, \frac{126}{2730}\times 3$ seen
		B1	$\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$ seen in numerator (SSS) **SCB1** $\frac{210}{2730}, \frac{1}{13}$ seen in numerator
		M1	Fraction with *their* **(c)** or correct in denominator $\left(\frac{720}{2730}, \frac{24}{91}, 0.263736\right)$
	$= \frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
	Alternative method for question 6(e)		
	$\frac{{}^{7}C_{2}\times{}^{3}C_{1}+{}^{7}C_{3}}{{}^{10}C_{3}}$	B1	${}^{7}C_{2}\times{}^{3}C_{1}$ seen (SSL, SLS, LSS) **SCB1** 21 × 3 seen or use of permutations
		B1	${}^{7}C_{3}$ seen in numerator (SSS) **SCB1** 35 seen in numerator or use of permutations
		M1	Fraction with ${}^{10}C_{3}$ or consistent with *their* numerator of **6(c)** in denominator
	$= \frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
		4	

196. 9709_s22_qp_51 Q: 4

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find $E(X)$. [1]

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

....

....

....

....

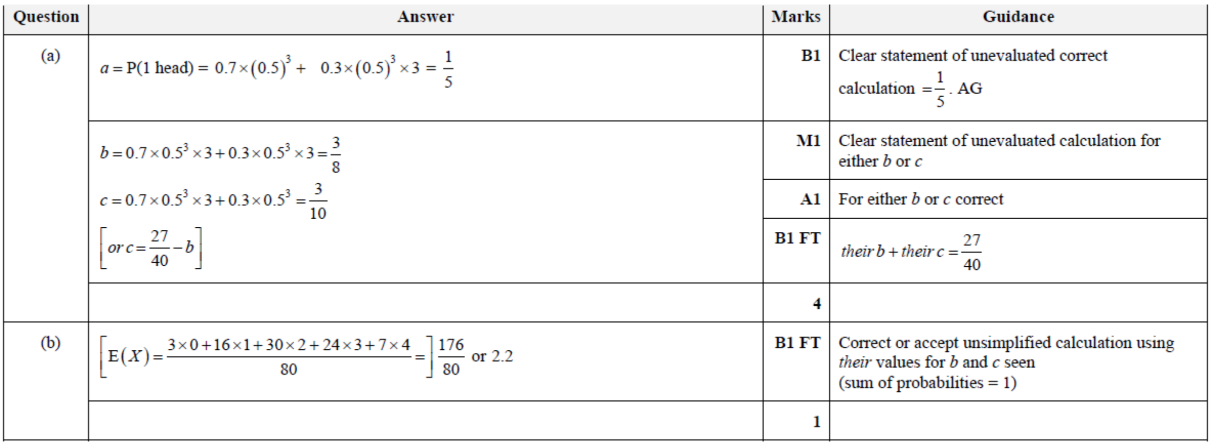
....

....

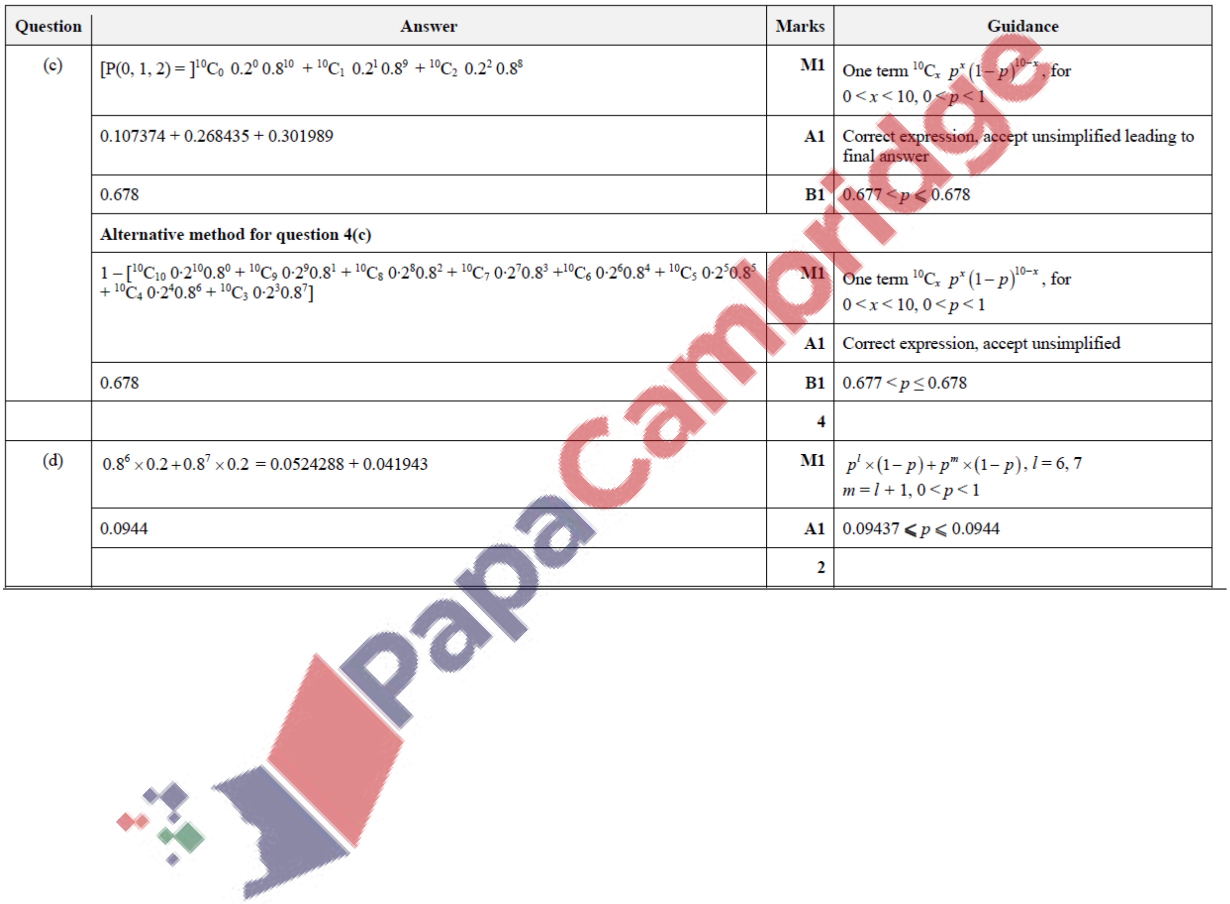
....

....

Answer:



Question	Answer	Marks	Guidance
(a)	$a = \text{P(1 head)} = 0.7\times(0.5)^3 + \;\; 0.3\times(0.5)^3\times 3 = \frac{1}{5}$	**B1**	Clear statement of unevaluated correct calculation $= \frac{1}{5}$. AG
	$b = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3\times 3 = \frac{3}{8}$ $c = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3 = \frac{3}{10}$ $\left[or\, c = \frac{27}{40} - b\right]$	**M1**	Clear statement of unevaluated calculation for either *b* or *c*
		A1	For either *b* or *c* correct
		B1 FT	$their\, b + their\, c = \frac{27}{40}$
		4	
(b)	$\left[\text{E}(X) = \frac{3\times 0+16\times 1+30\times 2+24\times 3+7\times 4}{80} = \right]\frac{176}{80}$ or 2.2	**B1 FT**	Correct or accept unsimplified calculation using *their* values for *b* and *c* seen (sum of probabilities = 1)
		1	



Question	Answer	Marks	Guidance
(c)	$[\text{P}(0, 1, 2) =]\,{}^{10}\text{C}_0\; 0.2^0\, 0.8^{10} + {}^{10}\text{C}_1\; 0.2^1\, 0.8^9 + {}^{10}\text{C}_2\; 0.2^2\, 0.8^8$	**M1**	One term ${}^{10}\text{C}_x\; p^x(1-p)^{10-x}$, for $0 < x < 10,\ 0 < p < 1$
	$0.107374 + 0.268435 + 0.301989$	**A1**	Correct expression, accept unsimplified leading to final answer
	0.678	**B1**	$0.677 < p \leqslant 0.678$
	Alternative method for question 4(c)		
	$1 - [{}^{10}\text{C}_{10}\, 0{\cdot}2^{10}0.8^0 + {}^{10}\text{C}_9\, 0{\cdot}2^9 0.8^1 + {}^{10}\text{C}_8\, 0{\cdot}2^8 0.8^2 + {}^{10}\text{C}_7\, 0{\cdot}2^7 0.8^3 + {}^{10}\text{C}_6\, 0{\cdot}2^6 0.8^4 + {}^{10}\text{C}_5\, 0{\cdot}2^5 0.8^5 + {}^{10}\text{C}_4\, 0{\cdot}2^4 0.8^6 + {}^{10}\text{C}_3\, 0{\cdot}2^3 0.8^7]$	**M1**	One term ${}^{10}\text{C}_x\; p^x(1-p)^{10-x}$, for $0 < x < 10,\ 0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	**B1**	$0.677 < p \le 0.678$
		4	
(d)	$0.8^6\times 0.2 + 0.8^7\times 0.2 = 0.0524288 + 0.041943$	**M1**	$p^l\times(1-p) + p^m\times(1-p),\ l = 6, 7$ $m = l + 1,\ 0 < p < 1$
	0.0944	**A1**	$0.09437 \leqslant p \leqslant 0.0944$
		2	

197. 9709_s22_qp_52 Q: 2

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

....

....

....

....

....

....

....

....

....

....

....

(b) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

....

....

....

....

....

....

....

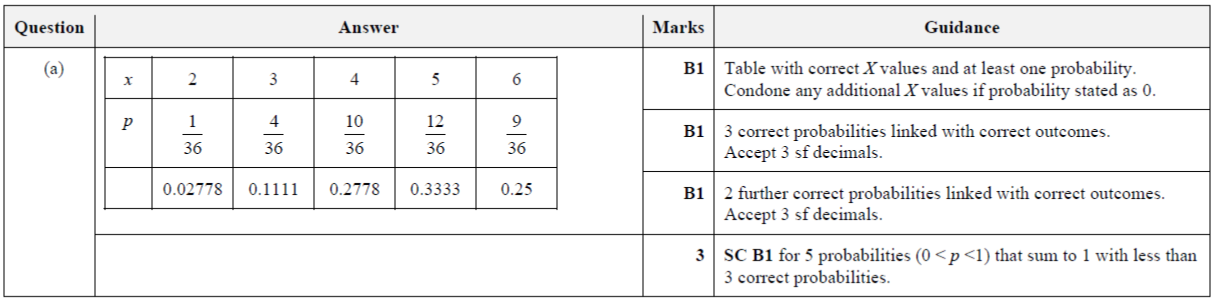
....

....

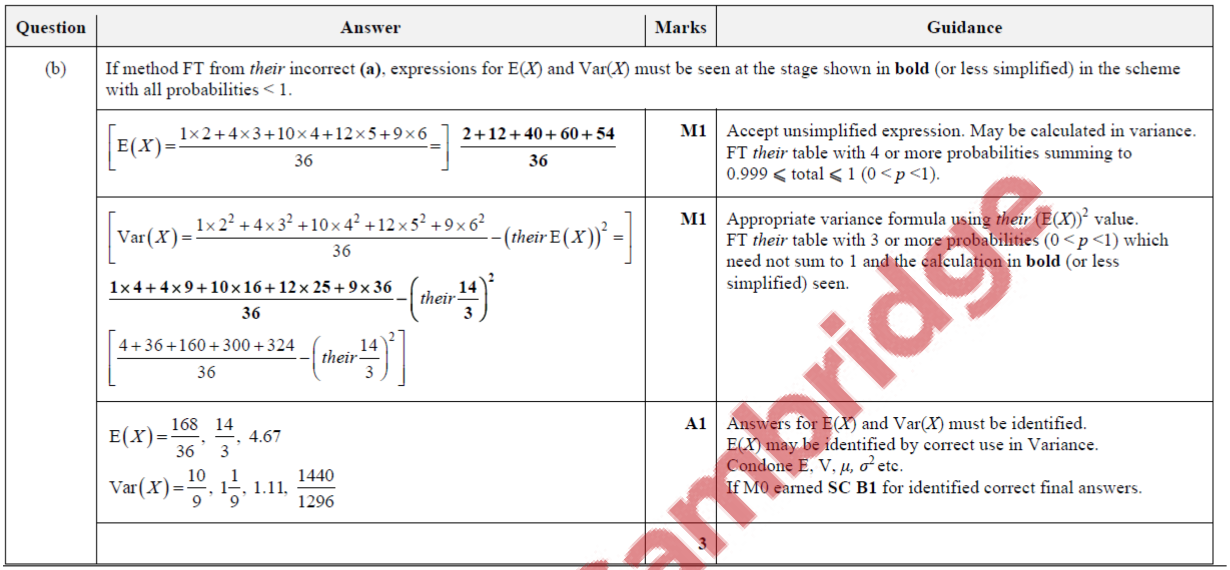
....

....

Answer:



<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">(a)</td><td rowspan="3"><table>
<tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>p</td><td>$\frac{1}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{9}{36}$</td></tr>
<tr><td></td><td>0.02778</td><td>0.1111</td><td>0.2778</td><td>0.3333</td><td>0.25</td></tr>
</table></td><td>B1</td><td>Table with correct X values and at least one probability.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>3 correct probabilities linked with correct outcomes.
Accept 3 sf decimals.</td></tr>
<tr><td>B1</td><td>2 further correct probabilities linked with correct outcomes.
Accept 3 sf decimals.</td></tr>
<tr><td></td><td></td><td>3</td><td>SC B1 for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.</td></tr>
</table>



Question	Answer	Marks	Guidance
(b)	If method FT from *their* incorrect **(a)**, expressions for E(X) and Var(X) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.		
	$\left[\mathrm{E}(X)=\frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36}=\right]$ $\boldsymbol{\frac{2+12+40+60+54}{36}}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to $0.999 \leqslant$ total $\leqslant 1$ ($0 < p < 1$).
	$\left[\mathrm{Var}(X)=\frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36}-(\textit{their}\,\mathrm{E}(X))^2=\right]$ $\boldsymbol{\frac{1\times4+4\times9+10\times16+12\times25+9\times36}{36}-\left(\textit{their}\,\frac{14}{3}\right)^2}$ $\left[\frac{4+36+160+300+324}{36}-\left(\textit{their}\,\frac{14}{3}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
	$\mathrm{E}(X)=\frac{168}{36},\ \frac{14}{3},\ 4.67$ $\mathrm{Var}(X)=\frac{10}{9},\ 1\frac{1}{9},\ 1.11,\ \frac{1440}{1296}$	**A1**	Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
		3	

198. 9709_s22_qp_53 Q: 3

The random variable X takes the values -2, 1, 2, 3. It is given that $\mathrm{P}(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$k = \frac{1}{18}$ $(4k + k + 4k + 9k = 18k = 1)$	**B1**	SOI
	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & -2 & 1 & 2 & 3 \\ \hline P(X=x) & \frac{4}{18} & \frac{1}{18} & \frac{4}{18} & \frac{9}{18} \\ \hline \end{array}$	**M1**	Table with correct x values and at least one probability accurate using *their* k. Values need not be in order, lines may not be drawn, may be vertical, x and $P(X=x)$ may be omitted. Condone any additional X values if probability stated as 0.
		A1	Remaining probabilities correct.
		3	

Question	Answer	Marks	Guidance
(b)	$\left[E(X) = \frac{4\times -2 + 1\times 1 + 4\times 2 + 9\times 3}{18} = \right]$ $\frac{-8+1+8+27}{18}$	**M1**	$-8k + k + 8k + 27k$ May be implied by use in Variance. Accept unsimplified expression. FT *their* table if probabilities sum to 1 or 0.999. **SC B1** $28k$.
	$\left[\mathrm{Var}(X) = \frac{4\times(-2)^2 + 1\times 1^2 + 4\times 2^2 + 9\times 3^2}{18} - (\textit{their}\, E(X))^2 = \right]$ $= \frac{16+1+16+81}{18} - \left(\textit{their}\, \frac{28}{18}\right)^2$	**M1**	$16k + k + 16k + 81k - (\textit{their}\text{ mean})^2$ FT *their* table even if probabilities not summing to 1. Note: If table is correct, $\frac{114}{18} - (\textit{their}\, E(X))^2$ M1. **SC B1** $114k - (\textit{their}\text{ mean})^2$.
	$E(X) = \frac{14}{9}, 1\frac{5}{9}, 1.56$, $\mathrm{Var}(X) = \frac{317}{81}, 3\frac{74}{81}, 3.91$	**A1**	Answers for $E(X)$ and $\mathrm{Var}(X)$ must be identified. $3.91 \leqslant \mathrm{Var}(X) \leqslant 3.914$
		3	

199. 9709_s22_qp_53 Q: 4

Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

....

....

....

....

(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

....

....

....

....

Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	$\left[\left(\frac{5}{6}\right)^7 \times \frac{1}{6} = \right] 0.0465, \frac{78125}{1679616}$	**B1**	$0.0465 \leqslant p < 0.04652$
		1	
(b)	$P(X < 6) = 1 - \left(\frac{5}{6}\right)^5$ or $\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	**M1**	$1 - p^n$, $0 < p < 1$, $n = 4, 5, 6$ or sum of 4, 5 or 6 terms $p \times (1-p)^n$ for $n = 0,1,2,3,4(5)$.
	$0.598, \frac{4651}{7776}$	**A1**	
		2	
(c)	[Probability of total less than 4 is] $\frac{3}{36}$ or $\frac{1}{12}$	**B1**	SOI
	[1 – P(0, 1, 2)] $= 1 - ({}^{10}C_0 \left(\frac{1}{12}\right)^0 \left(\frac{11}{12}\right)^{10} + {}^{10}C_1 \left(\frac{1}{12}\right)^1 \left(\frac{11}{12}\right)^9 + {}^{10}C_2 \left(\frac{1}{12}\right)^2 \left(\frac{11}{12}\right)^8)$	**M1**	One term ${}^{10}C_x \; p^x (1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$.
	$1 - (0.418904 + 0.380822 + 0.155791)$	**A1 FT**	Correct expression. Accept unsimplified.
	0.0445	**A1**	$0.04448 \leqslant p \leqslant 0.0445$
		4	

200. 9709_m21_qp_52 Q: 1

A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.

(a) Find the probability that a score of 3 is obtained for the first time on the 8th spin. [1]

(b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	$\left[\left(\frac{4}{5}\right)^7 \frac{1}{5} =\right]$ $\frac{16384}{390625}$ or 0·0419[43…]	**B1**	Evaluated, final answer.
		1	
(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1 - p^{n}$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$ Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5 + {}^5C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right) + {}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 + {}^5C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 + {}^5C_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5 + {}^5C_4(p)^4(q) + {}^5C_3(p)^3(q)^2 + {}^5C_2(p)^2(q)^3 + {}^5C_1(p)(q)^4$ or $(p)^6 + {}^6C_5(p)^5(q) + {}^6C_4(p)^4(q)^2 + {}^6C_3(p)^3(q)^3$ $+{}^6C_2(p)^2(q)^4 + {}^6C_1(p)(q)^5,\ 0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
		2	

201. 9709_m21_qp_52 Q: 4

The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5-x)$, where k is a constant.

(a) Draw up the probability distribution table for X, in terms of k. [2]

....

....

....

....

....

....

....

....

(b) Show that $\text{Var}(X) = 1.05$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 \\ \hline \text{prob} & 4k & 6k & 6k & 4k \\ \hline\end{array}$	**B1**	Table with × values and one correct probability expressed in terms of k. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
(b)	$[4k+6k+6k+4k=1]\ k=\frac{1}{20}\quad(=0.05)$	**B1**	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X)=1\times\frac{4}{20}+2\times\frac{6}{20}+3\times\frac{6}{20}+4\times\frac{4}{20}\ =\frac{4}{20}+\frac{12}{20}+\frac{18}{20}+\frac{16}{20}$ $(=2.5)$	**M1**	Accept unsimplified expression. Condone $4k+12k+18k+16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\mathrm{Var}(X)=1^2\times\frac{4}{20}+2^2\times\frac{6}{20}+3^2\times\frac{6}{20}+4^2\times\frac{4}{20}-\left(\mathit{their}\,2\frac{1}{2}\right)^2$ $=(4+24+54+64)\times\mathit{their}\,0.05-(\mathit{their}\,2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20}+(2-2.5)^2\times\frac{6}{20}\ +(3-2.5)^2\times\frac{6}{20}+(4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(E(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	**A1**	AG, NFWW.
		4	

202. 9709_s21_qp_51 Q: 7

Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $\mathrm{P}(X = 3) = \frac{6}{35}$. [2]

....

....

....

....

....

....

(b) Draw up the probability distribution table for X. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Find Var(X). [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$P(X=3)=\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$	M1	$\frac{m}{7}\times\frac{n}{6}\times\frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

Question	Answer	Marks	Guidance
(b)	$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 & 5 \\ \hline p & \frac{15}{35} & \frac{10}{35} & \frac{6}{35} & \frac{3}{35} & \frac{1}{35} \\ \hline\end{array}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
(c)	$[E(X) = 1\times\frac{15}{35}+2\times\frac{10}{35}+3\times\frac{6}{35}+4\times\frac{3}{35}+5\times\frac{1}{35}]$ $E(X)=\frac{15+20+18+12+5}{35}\left[=\frac{70}{35}=2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\operatorname{Var}(X)=\left[\frac{1^2\times15+2^2\times10+3^2\times6+4^2\times3+5^2\times1}{35}-2^2=\right]$ $\frac{15+40+54+48+25}{35}-2^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[=\frac{182}{35}-4\right]=\frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

203. 9709_s21_qp_52 Q: 1

An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. [1]

....

....

....

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

....

....

....

....

....

....

....

....

(c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	6	B1	WWW
		1	
(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p)+p^4(1-p)+p^5(1-p)+p^6(1-p),\ 0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3-\left(\frac{5}{6}\right)^7$	M1	$p^3-p^7,\ 0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
(c)	$1-\left(\frac{5}{6}\right)^9$	M1	$1-p^n,\ 0<p<1,\ n=9, 10$
	0.806	A1	
	Alternative method for Question 1(c)		
	$\frac{1}{6}+\frac{1}{6}\left(\frac{5}{6}\right)+\frac{1}{6}\left(\frac{5}{6}\right)^2+\cdots+\frac{1}{6}\left(\frac{5}{6}\right)^8$	M1	$p+p(1-p)+p(1-p)^2+p(1-p)^3+p(1-p)^4+p(1-p)^5+p(1-p)^6+p(1-p)^7+p(1-p)^8(+p(1-p)^9),\ 0<p<1$ As per answer for minimum terms shown
	0.806	A1	
		2	

204. 9709_s21_qp_52 Q: 4

A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $\text{Var}(X)$. [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">(a)</td><td rowspan="3"><table><tr><td>X</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X)$</td><td>$\frac{1}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{1}{9}$</td><td>$\frac{3}{9}$</td><td>$\frac{2}{9}$</td></tr></table></td><td>B1</td><td>Table with correct X values and at least one probability
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>2 correct probabilities linked with correct outcomes, may not be in table.</td></tr>
<tr><td>B1</td><td>3 further correct probabilities linked with correct outcomes, may not be in table.
SC if less than 2 correct probabilities seen, award
SCB1 for sum of *their* 4 or 5 probabilities in table = 1</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">(b)</td><td>$\left[E(X)=\frac{-1\times1+(0\times2)+1\times1+2\times3+3\times2}{9}=\right]$
$\frac{-1+1+6+6}{9}$</td><td>M1</td><td>May be implied by use in variance, accept unsimplified expression.
FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999</td></tr>
<tr><td>[Var (X) =]
$\left[\frac{-1^2\times1+(0^2\times2)+1^2\times1+2^2\times3+3^2\times2}{9}-(their\ E(X))^2\right]$
$\frac{1+0+1+12+18}{9}-(their\ E(X))^2$</td><td>M1</td><td>Appropriate variance formula using *their* $(E(X))^2$ value.
FT *their* table even if *their* 3 or more probabilities not summing to 1.</td></tr>
<tr><td>$E(X)=\frac{4}{3}$ or 1.33 and $Var(X)=\frac{16}{9}$ or 1.78</td><td>A1</td><td>Answers for E(X) and Var(X) must be identified</td></tr>
<tr><td></td><td>3</td><td>**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1</td></tr>
</table>

205. 9709_s21_qp_53 Q: 2

The random variable X can take only the values -2, -1, 0, 1, 2. The probability distribution of X is given in the following table.

x	-2	-1	0	1	2
$P(X = x)$	p	p	0.1	q	q

Given that $P(X \geqslant 0) = 3P(X < 0)$, find the values of p and q. [4]

Answer:

Question	Answer	Marks	Guidance
2	$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
	$0.1 + 2q = 3(2p)$	**B1**	Use given information
	Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
		4	

206. 9709_s21_qp_53 Q: 4

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(a) Find the probability that the score is 4 on a single throw of the three dice. [3]

(b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability = $\left(\frac{1}{6}\right)^3 \times 3$	**M1**	$\left(\frac{1}{6}\right)^3 \times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	**A1**	Accept $\frac{3}{216}$ or $0.013\dot{8}$ or 0.0139
		3	
(b)	$\text{P}(18) = \left(\frac{1}{6}\right)^3 \left[= \frac{1}{216}\right]$	**B1**	
	$\text{P(18 on 5th throw)} = \left(\frac{215}{216}\right)^4 \times \frac{1}{216}$	**M1**	$(1 - p)^4 p$, $0 < \textit{their } p < 1$
	0.00454	**A1**	
		3	

207. 9709_w21_qp_51 Q: 1

Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

....

....

....

....

....

....

....

....

....

....

....

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	**M1**	$(1-p)^6\, p,\ 0<p<1$
	0.0445, $\frac{729}{16384}$	**A1**	
		2	
(b)	$\left(\frac{3}{4}\right)^9$	**M1**	$\left(\frac{3}{4}\right)^n$ *or* $p^n,\ 0<p<1, n=8,9,10$
	0.0751, $\frac{19683}{262144}$	**A1**	
		2	

208. 9709_w21_qp_51 Q: 4

A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered -1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $\text{Var}(X)$. [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(a)</td><td><table><tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p</td><td>$\frac{1}{12}=0.0833$</td><td>$\frac{2}{12}=0.167$</td><td>$\frac{4}{12}=0.333$</td><td>$\frac{3}{12}=0.25$</td><td>$\frac{2}{12}=0.167$</td></tr></table></td><td>B1</td><td><table><tr><td></td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>– 1</td><td>–1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>3</td></tr></table>
Table with x values and at least one probability substituted, $0 < p < 1$.
Condone any additional x values if probability stated as 0.</td></tr>
<tr><td></td><td></td><td>B1</td><td>2 correct identified probabilities.</td></tr>
<tr><td></td><td></td><td>B1</td><td>All probabilities correct (accept to 3sf).
SC if less than 2 correct probabilities:
SC B1 4 or 5 probabilities summing to one.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>(b)</td><td>$\mathrm{E}(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$</td><td>M1</td><td>May be implied by use in Variance, accept unsimplified expression.
Probabilities must sum to 1 ± 0.001.</td></tr>
<tr><td></td><td>$\mathrm{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$</td><td>M1</td><td>Appropriate variance formula using <i>their</i> $(\mathrm{E}(X))^2$. FT accept probabilities not summing to 1.
Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ <i>or</i> $\frac{35}{12}-\frac{25}{9}$ from correct table.</td></tr>
<tr><td></td><td>$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48}$, 1.35</td><td>A1</td><td>WWW</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

209. 9709_w21_qp_52 Q: 3

A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X. [3]

....

....

....

....

....

....

....

....

(c) Find E(X). [1]

Answer:

Question	Answer	Marks	Guidance
(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	**M1**	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability × 3, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$, (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
	Alternative method for question 3(a)		
	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_3}$	**M1**	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_r}$, $r = 2, 3, 4$
		M1	$\frac{{}^5C_s\times{}^4C_t}{{}^9C_3}$, $s + t = 3$
	$\left[\frac{30}{84}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, WWW.
		3	

Question	Answer	Marks	Guidance
(b)	$\begin{array}{c\|c\|c\|c\|c} X & 0 & 1 & 2 & 3 \\ \hline P(X) & \frac{24}{504} \left[=\frac{1}{21}, 0.0476\right] & \frac{180}{504} \left[=\frac{5}{14}, 0.357\right] & \frac{240}{504} \left[=\frac{10}{21}, 0.476\right] & \frac{60}{504} \left[=\frac{5}{42}, 0.119\right] \end{array}$	**B1**	Table with correct *X* values and one correct probability inserted appropriately. Condone any additional *X* values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or *X* value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
(c)	[E(*X*) =] $\frac{840}{504}$, $\frac{5}{3}$, 1.67	**B1**	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

210. 9709_w21_qp_52 Q: 5

In a certain region, the probability that any given day in October is wet is 0.16, independently of other days.

(a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet. [3]

(b) Find the probability that the first wet day in October is 8 October. [2]

(c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	[P(0, 1, 2) =] $^{10}C_0\,0.16^0\,0.84^{10} + {}^{10}C_1\,0.16^1\,0.84^9 + {}^{10}C_2\,0.16^2\,0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	**M1**	One term: $^{10}C_x p^x\,(1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	**A1**	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
(b)	$(0.84)^7\,0.16$	**M1**	$(1-p)^7p$, $0 < p < 1$
	0.0472	**A1**	0.0472144 to at least 3sf.
		2	
(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	**M1**	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	**A1**	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

211. 9709_w21_qp_53 Q: 6

In a game, Jim throws three darts at a board. This is called a 'turn'. The centre of the board is called the bull's-eye.

The random variable X is the number of darts in a turn that hit the bull's-eye. The probability distribution of X is given in the following table.

x	0	1	2	3
$P(X = x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$.

(a) Find the values of p and q. [4]

....

....

....

....

....

....

....

....

....

....

(b) Find $\text{Var}(X)$. [2]

....

....

....

....

....

....

....

....

Jim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the probability that $X = 1$ in at least 3 of 12 randomly chosen turns. [3]

(d) Find the probability that Jim first succeeds in hitting the bull's-eye with all three darts on his 9th turn. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	$p + q + 0.65 = 1$	**B1**	Sum of probabilities = 1.
	$p + 2q + 0.15 = 0.55$	**B1**	Use given information.
	Solve 2 linear equations	**M1**	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p = 0.3, \frac{3}{10}, \quad q = 0.05, \frac{1}{20}$	**A1**	CAO, both WWW If **M0** with correct answers **SC B1**.
		4	
(b)	Var $(X) =$ *their* $0.3 + 4 \times$ *their* $0.05 + 9 \times 0.05 - 0.55^2$	**M1**	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475 \left[\frac{259}{400}\right]$	**A1**	CAO (must be exact).
		2	
(c)	$1 - P(0, 1, 2) = 1 - ({}^{12}C_0\ 0.3^0\ 0.7^{12} + {}^{12}C_1\ 0.3^1\ 0.7^{11} + {}^{12}C_2\ 0.3^2\ 0.7^{10})$	**M1**	One correct term: ${}^{12}C_x p^x (1 - p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	$1 - (0.01384 + 0.07118 + 0.16779)$	**A1FT**	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to **FT** *their p* from **6(a)** or correct.
	0.747	**A1**	
		3	
(d)	$(0.95)^8 \times 0.05 = 0.0332$ or $0.95^8 - 0.95^9 = 0.0332$	**B1**	Evaluated.
		1	

212. 9709_m20_qp_52 Q: 2

An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained.

(a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6. [3]

....

....

....

....

....

....

....

....

....

....

On another occasion, this die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the probability distribution table for X. [3]

(c) Find $\mathrm{E}(X)$. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$	**M1**	One correct term with $0 < p < 1$
	$=\frac{4}{27}+\frac{8}{81}+\frac{16}{243}\left(=\frac{2432}{7776}\right)$	**A1**	Correct expression, accept unsimplified
	$=\frac{76}{243}$ or 0.313	**A1**	
		3	

Question	Answer	Marks	Guidance
(b)	$\begin{array}{c\|c\|c\|c\|c} x & 0 & 1 & 2 & 3 \\ \hline \mathrm{P}(x) & \frac{8}{27} & \frac{12}{27} & \frac{6}{27} & \frac{1}{27} \end{array}$	**B1**	Probability distribution table with correct values of x, no additional values unless with probability of 0 stated, at least one non-zero probability included
	$\mathrm{P}(0)=\left(\frac{2}{3}\right)^3$ $\mathrm{P}(1)=\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2\times 3$ $\mathrm{P}(2)=\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2\times 3$ $\mathrm{P}(3)=\left(\frac{1}{3}\right)^3$	**B1**	1 correct probability seen (may not be in table) **or** 3 or 4 non-zero probabilities summing to 1
		B1	All probabilities correct
		3	
(c)	$\mathrm{E}(X)=\left[0\times\frac{8}{27}\right]+1\times\frac{12}{27}+2\times\frac{6}{27}+3\times\frac{1}{27}$ $=\left[\frac{0}{27}\right]+\frac{12}{27}+\frac{12}{27}+\frac{3}{27}$	**M1**	Correct method from *their* probability distribution table with at least 3 terms, $0 \leqslant$ *their* $\mathrm{P}(x) \leqslant 1$, accept unsimplified
	$= 1$	**A1**	
		2	

213. 9709_s20_qp_51 Q: 1

The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

....

....

....

....

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the mean of X. [1]

....

....

....

....

(c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

....

....

....

....

(d) Find $P(X < 8)$. [2]

....

....

....

....

....

....

Answer:

Question	Answer	Marks
(a)	Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36}=\frac{1}{12}$ **AG**	**B1**
		1
(b)	Mean = $\frac{1}{\frac{1}{12}}$ = 12	**B1**
		1
(c)	$\left(\frac{11}{12}\right)^5\times\frac{1}{12}=0.0539$ or $\frac{161051}{2985984}$	**B1**
		1
(d)	$1-\left(\frac{11}{12}\right)^7$	**M1**
	0.456 or $\frac{16344637}{35831808}$	**A1**
		2

214. 9709_s20_qp_51 Q: 3

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>(a)</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{1}{56}$</td><td>$\frac{15}{56}$</td><td>$\frac{30}{56}$</td><td>$\frac{10}{56}$</td></tr></table>(B1 for probability distribution table with correct outcome values)</td><td>B1</td></tr>
<tr><td></td><td>$\mathrm{P}(0)=\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}=\frac{1}{56}$
$\mathrm{P}(1)=\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3=\frac{15}{56}$
$\mathrm{P}(2)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times 3=\frac{30}{56}$
$\mathrm{P}(3)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{10}{56}$
(M1 for denominator 8×7×6)</td><td>M1</td></tr>
<tr><td></td><td>Any one probability correct (with correct outcome)</td><td>A1</td></tr>
<tr><td></td><td>All probabilities correct</td><td>A1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>(b)</td><td>$1-\mathrm{P}(8,9,10)=1-\left[{}^{10}\mathrm{C}_8\,0.64^8 0.36^2+{}^{10}\mathrm{C}_9\,0.64^9 0.36^1+0.64^{10}\right]$</td><td>M1</td></tr>
<tr><td></td><td>$1-(0.164156+0.064852+0.11529)$</td><td>M1</td></tr>
<tr><td></td><td>0.759</td><td>A1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>

215. 9709_s20_qp_52 Q: 5

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

(b) Draw up the probability distribution table for X. [3]

....

....

....

....

....

....

(c) Find E(X) and Var(X). [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>(a)</td><td><table><tr><th></th><th>1</th><th>1</th><th>2</th><th>2</th><th>3</th></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr><tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr></table></td><td>M1</td></tr>
<tr><td></td><td>$\frac{7}{15}$ AG</td><td>A1</td></tr>
<tr><td></td><td></td><td>2</td></tr>
<tr><td>(b)</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{2}{15}$</td><td>$\frac{6}{15}$</td><td>$\frac{7}{15}$</td></tr></table></td><td>B1</td></tr>
<tr><td></td><td>P(1) or P(2) correct</td><td>B1</td></tr>
<tr><td></td><td>3rd probability correct, FT sum to 1</td><td>B1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>(c)</td><td>$\mathrm{E}(X)=\frac{2+12+21}{15}=\frac{35}{15}=\frac{7}{3}$</td><td>B1</td></tr>
<tr><td></td><td>$\mathrm{Var}(X)=\frac{1^2\times2+2^2\times6+3^2\times7}{15}-\left(\frac{7}{3}\right)^2$</td><td>M1</td></tr>
<tr><td></td><td>$\frac{22}{45}(0.489)$</td><td>A1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

216. 9709_s20_qp_53 Q: 2

In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car. [1]

....

....

....

....

....

(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks
(a)	$0.22^3 = 0.0106$	**B1**
		1
(b)	$P(2, 3, 4) = {}^{16}C_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	**M1**
	$0.179205 + 0.235877 + 0.216221$	**A1**
	0.631	**A1**
		3

217. 9709_s20_qp_53 Q: 4

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

(b) Find Var(X). [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="4">(a)</td><td>
<table>
<tr><td>–1</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>2</td></tr>
<tr><td>2</td><td>3</td><td>3</td><td>4</td></tr>
</table>
<table>
<tr><td>x</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table>
Probability distribution table with correct scores with at least one probability</td><td>B1</td></tr>
<tr><td>At least 4 probabilities correct</td><td>B1</td></tr>
<tr><td>All probabilities correct</td><td>B1</td></tr>
<tr><td></td><td>3</td></tr>
<tr><td rowspan="4">(b)</td><td>$\mathrm{E}(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$</td><td>B1</td></tr>
<tr><td>$\mathrm{Var}(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$</td><td>M1</td></tr>
<tr><td>$\frac{37}{18}$ (= 2.06)</td><td>A1</td></tr>
<tr><td></td><td>3</td></tr>
</table>

218. 9709_w20_qp_51 Q: 3

Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

(b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

Answer:

(a)	$P(X > 6) = 0.75^6$	**M1**	p^n, $n = 6, 7$ $\;0 < p < 1$
	$0.178, \dfrac{729}{4096}$	**A1**	0·17797…
		2	

Question	Answer	Marks	Guidance
(b)	$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\ 0.25^1\, 0.75^9 + {}^{10}C_2\, 0.25^2\, 0.75^8)$	**M1**	Binomial term of form ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$
	1 – (0·0563135 + 0·1877117 + 0·2815676)	**A1**	Correct unsimplified expression
	0·474	**A1**	$0{\cdot}474 \leqslant p \leqslant 0{\cdot}4744$
		3	

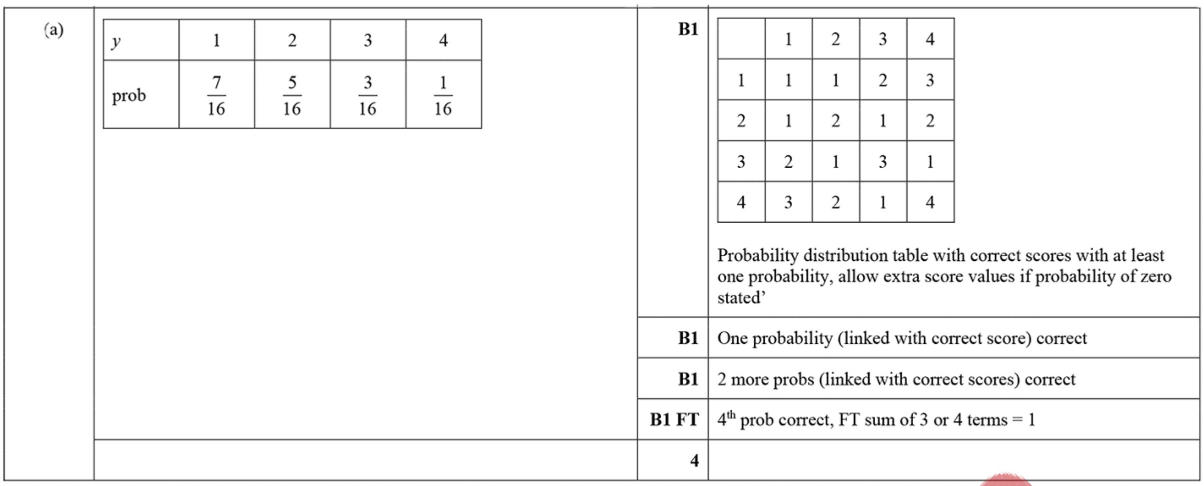
219. 9709_w20_qp_51 Q: 4

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

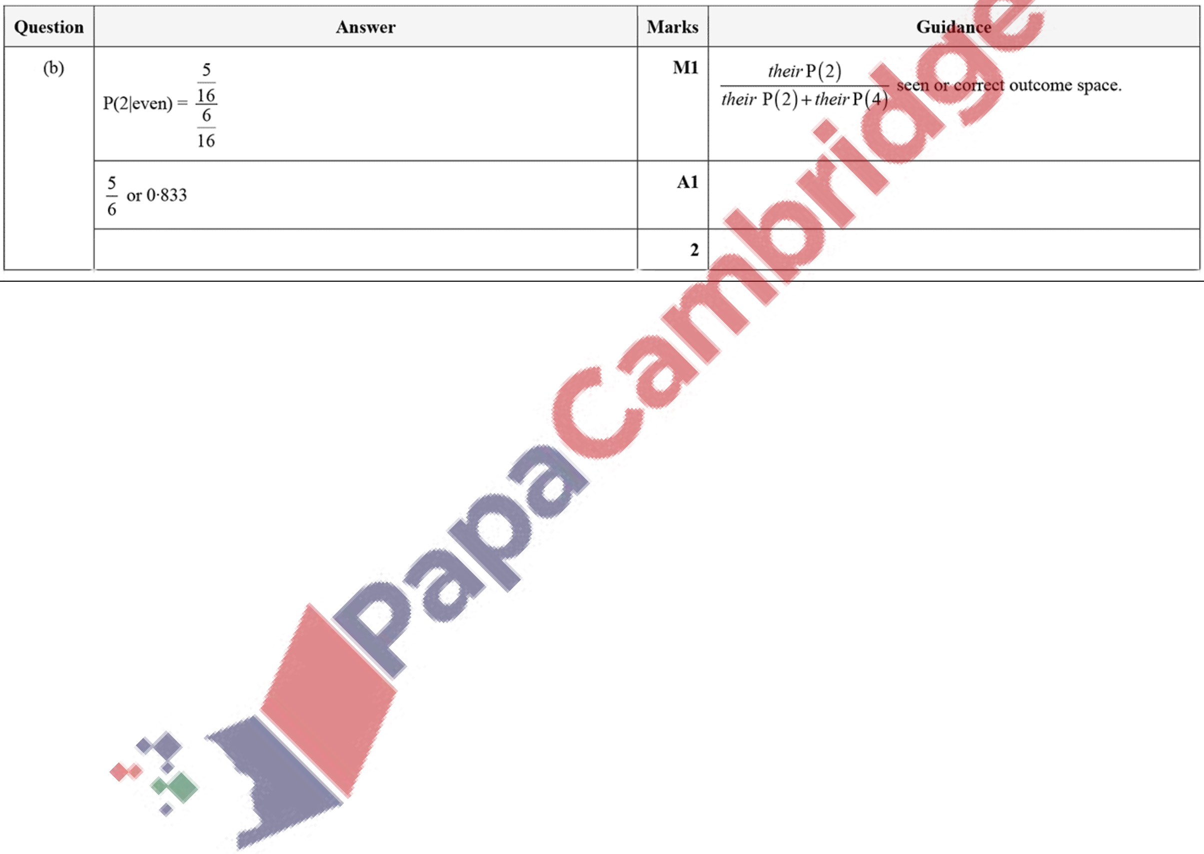
(a) Draw up the probability distribution table for Y. [4]

(b) Find the probability that $Y = 2$ given that Y is even. [2]

Answer:



<table>
<tr><td>(a)</td><td><table><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>prob</td><td>$\frac{7}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{1}{16}$</td></tr></table></td><td>B1</td><td><table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>2</td><td>1</td><td>2</td><td>1</td><td>2</td></tr><tr><td>3</td><td>2</td><td>1</td><td>3</td><td>1</td></tr><tr><td>4</td><td>3</td><td>2</td><td>1</td><td>4</td></tr></table>Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated’</td></tr>
<tr><td></td><td></td><td>B1</td><td>One probability (linked with correct score) correct</td></tr>
<tr><td></td><td></td><td>B1</td><td>2 more probs (linked with correct scores) correct</td></tr>
<tr><td></td><td></td><td>B1 FT</td><td>4th prob correct, FT sum of 3 or 4 terms = 1</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>



Question	**Answer**	**Marks**	**Guidance**
(b)	P(2|even) = $\frac{\frac{5}{16}}{\frac{6}{16}}$	**M1**	$\frac{their\ \text{P}(2)}{their\ \text{P}(2)+their\ \text{P}(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or 0·833	**A1**	
		2	

220. 9709_w20_qp_52 Q: 1

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

....

....

....

....

....

....

....

....

....

....

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2+pq^3+ pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$
	$0{\cdot}598$, $\frac{4651}{7776}$	**A1**	
		2	
(b)	$(1 - P(0, 1, 2))$ $1-\left(\left(\frac{5}{6}\right)^{10}+{}^{10}C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9+{}^{10}C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	**M1**	${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0,10$
	$1 - (0{\cdot}1615056 + 0{\cdot}3230111 + 0{\cdot}290710)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket
	$0{\cdot}225$	**A1**	$0{\cdot}2247 < p \leq 0{\cdot}225$, WWW
		3	

221. 9709_w20_qp_52 Q: 2

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Given that $\mathrm{E}(X) = \frac{15}{8}$, find $\mathrm{Var}(X)$. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3$	**M1**	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times3$, $1\leqslant a,b,c\leqslant5$, $d,e,f\leqslant8$, a, b, c, d, e, f, k all integers.$1<k\leqslant3$,
	$\frac{15}{56}$	**A1**	AG, WWW
	Alternative method for question 2(a)		
	$\frac{{}^5C_1\times{}^3C_2}{{}^8C_3}$	**M1**	$\frac{{}^aC_1\times{}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e\left(or\ {}^aC_1\times{}^bC_2\right)}{{}^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a+b=8$, $d+e=3$
	$\frac{15}{56}$	**A1**	AG, WWW, $\frac{15}{56}$ must be seen
		2	
(b)	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & 0 & 1 & 2 & 3 \\ \hline \text{Prob.} & \frac{1}{56} & \frac{15}{56} & \frac{30}{56}=\frac{15}{28} & \frac{10}{56}=\frac{5}{28} \\ \hline & 0{\cdot}0179 & 0{\cdot}268 & 0{\cdot}536 & 0{\cdot}179 \\ \hline \end{array}$	**B1**	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	

Question	Answer	Marks	Guidance
(c)	$\text{Var}(X)=\frac{(0^2\times1)+1^2\times15+2^2\times30+3^2\times10}{56}-\left(\frac{15}{8}\right)^2$ $=\frac{15}{56}+\frac{120}{56}+\frac{90}{56}-\left(\frac{15}{8}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have ' – mean^2' (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\frac{225}{448}$, 0·502	**A1**	
		2	

222. 9709_w20_qp_53 Q: 2

An ordinary fair die is thrown until a 6 is obtained.

(a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X. [1]

(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	$\left(\frac{5}{6}\right)^8$	M1	p^8, $0 < p < 1$, no x, + or -
	0.233	A1	
		2	
(b)	36	B1	
		1	
(c)	$\mathrm{P}(X=10) + \mathrm{P}(X=11) = \left(\frac{35}{36}\right)^9 \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \frac{1}{36}$	M1	OE, unsimplified expression in form $p^9q + p^{10}q$, $p + q = 1$, no ×
	0.0425	A1	
		2	

223. 9709_w20_qp_53 Q: 6

Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

(c) Given that $\mathrm{E}(X) = \frac{32}{15}$, find $\mathrm{Var}(X)$. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	**M1**	One 3 factor probability with 3, 3, 5 as denominators
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total = $\frac{20}{45}=\frac{4}{9}$	**A1**	AG, Total of 3 products with clear context
		3	
(b)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline x & 0 & 1 & 2 & 3 \\ \hline \text{Prob.} & \frac{1}{45} & \frac{8}{45} & \frac{20}{45} & \frac{16}{45} \\ \hline\end{array}$	**B1**	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated’
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
(c)	$\text{Var}(X)=\frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45}-\left(\frac{32}{15}\right)^2$ $=\frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have ‘– mean^2’ (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	**A1**	
		2	

224. 9709_m19_qp_62 Q: 4

The random variable X takes the values -1, 1, 2, 3 only. The probability that X takes the value x is kx^2, where k is a constant.

(i) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td><table><tr><td>x</td><td>-1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p</td><td>k</td><td>k</td><td>$4k$</td><td>$9k$</td></tr></table></td><td>B1</td><td>Probability distribution table with correct values of <i>x</i>, no additional values unless with probability 0 stated, at least one correct probability including <i>k</i></td></tr>
<tr><td></td><td>$15k = 1$,</td><td>M1</td><td>Equating $\Sigma p = 1$, may be implied by answer</td></tr>
<tr><td></td><td>$k = \frac{1}{15}$</td><td>A1</td><td>If 0 scored, SCB2 for probability distribution table with correct numerical probabilities.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	$\mathrm{E}(X) = 8k + 27k = 35k = \frac{35}{15} = \frac{7}{3}$	**B1FT**	FT if 0< *their k*<1
	$\mathrm{Var}(X) = (k + k + 16k + 81k) - (35k)^2$	**M1**	Correct formula for variance, in terms of *k* at least – must have '– mean^2'(ft).
	$= 1.16\ ,\ \frac{52}{45}$	**A1**	
	Method 2		
	$\mathrm{E}(X) = \frac{8}{15} + \frac{27}{15} = \frac{35}{15} = \frac{7}{3}$	**B1FT**	FT if 0< *their k*<1
	$\mathrm{Var}(X) = \frac{1}{15} + \frac{1}{15} + \frac{16}{15} + \frac{81}{15} - \left(\frac{7}{3}\right)^2$	**M1**	Subst *their* values in correct var formula – must have '– mean^2'(ft) (condone probs not summing to exactly 1)
	= 1.16 (= 52/45)	**A1**	Using their values from **(i)**
		3	

225. 9709_s19_qp_61 Q: 6

At a funfair, Amy pays \$1 for two attempts to make a bell ring by shooting at it with a water pistol.

- If she makes the bell ring on her first attempt, she receives \$3 and stops playing. This means that overall she has gained \$2.
- If she makes the bell ring on her second attempt, she receives \$1.50 and stops playing. This means that overall she has gained \$0.50.
- If she does not make the bell ring in the two attempts, she has lost her original \$1.

The probability that Amy makes the bell ring on any attempt is 0.2, independently of other attempts.

(i) Show that the probability that Amy loses her original \$1 is 0.64. [2]

(ii) Complete the probability distribution table for the amount that Amy gains. [4]

Amy's gain ($)			
Probability	0.64		

(iii) Calculate Amy's expected gain. [1]

Answer:

Question	Answer	Marks	Guidance
(i)	P(loses \$1) = P(F and F) = 0.8 × 0.8	**M1**	0.8 x 0.8 or (1 – 0.2)(1-0.2) or P(F) × P(F) or P(F)+P(F) seen or implied
	= 0.64 AG	**A1**	Must see probabilities multiplied together with final answer and a clear probability statement or implied by labelled tree diagram
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">(ii)</td><td rowspan="5"><table><tr><td>Amount gained ($)</td><td>–1</td><td>0.50</td><td>2</td></tr><tr><td>Prob</td><td></td><td>0.16</td><td>0.2</td></tr></table></td><td>B1</td><td>–1 linked with 0.64 in table</td></tr>
<tr><td>B1</td><td>0.5 seen in table</td></tr>
<tr><td>B1</td><td>0.16 seen in table linked to their 0.5</td></tr>
<tr><td>B1</td><td>FT P(2.00 gained) = 0.36 – P(0.50 gained) or correct, and all amount gained linked correctly in table</td></tr>
<tr><td>4</td><td></td></tr>
<tr><td rowspan="2">(iii)</td><td>E(winnings) = –1 × 0.64 + 0.5 × 0.16 + 2 × 0.2
= –($)0.16, –16 cents</td><td>B1</td><td>FT Accept ($)0.16 or 16 cents loss. FT unsimplified E(winnings) from their table provided Σ<i>p</i> = 1</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

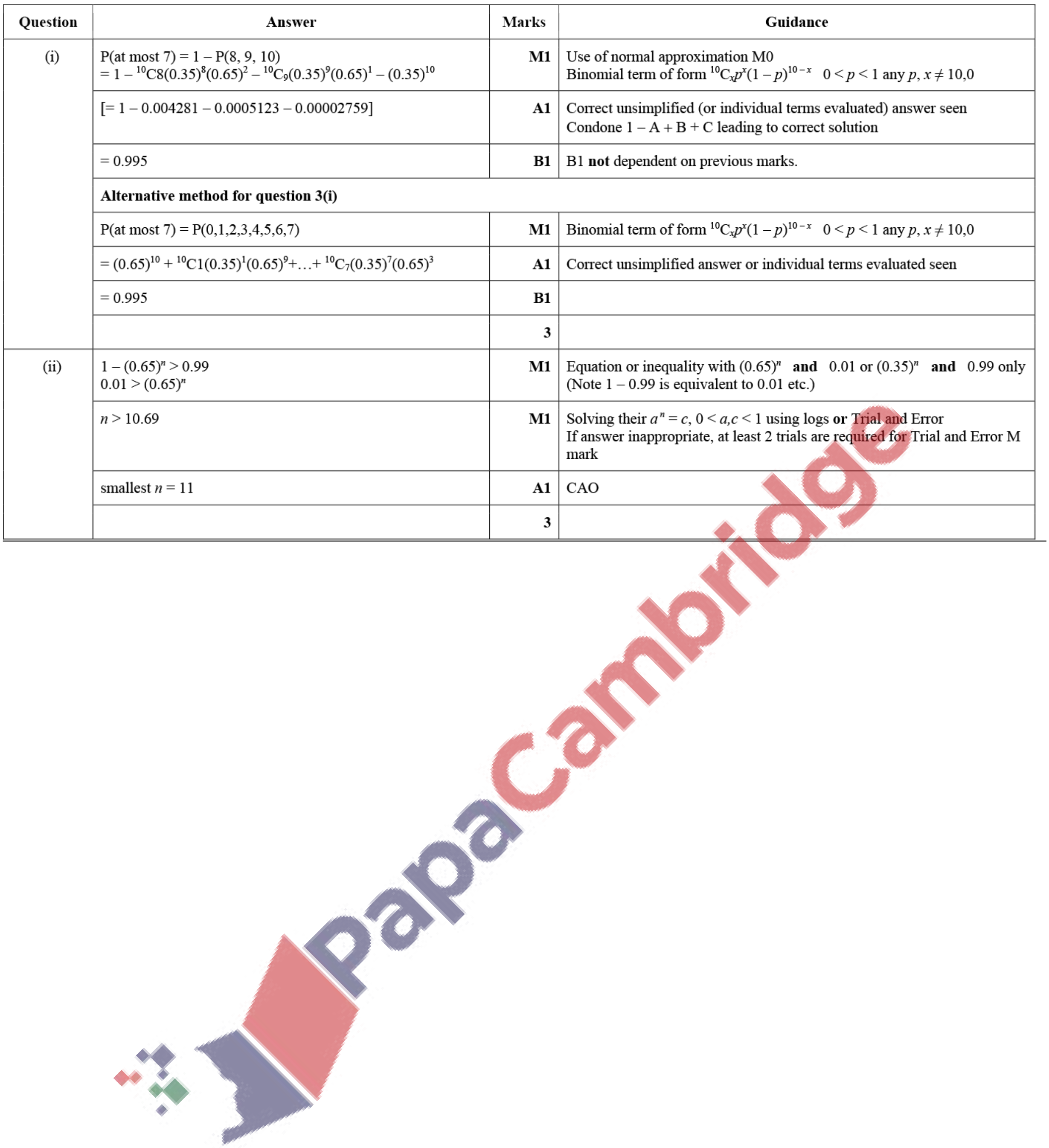
226. 9709_s19_qp_62 Q: 3

The probability that Janice will buy an item online in any week is 0.35. Janice does not buy more than one item online in any week.

(i) Find the probability that, in a 10-week period, Janice buys at most 7 items online. [3]

(ii) The probability that Janice buys at least one item online in a period of n weeks is greater than 0.99. Find the smallest possible value of n. [3]

Answer:



Question	Answer	Marks	Guidance
(i)	P(at most 7) = 1 – P(8, 9, 10) $= 1 - {}^{10}C8(0.35)^8(0.65)^2 - {}^{10}C_9(0.35)^9(0.65)^1 - (0.35)^{10}$	**M1**	Use of normal approximation M0 Binomial term of form ${}^{10}C_xp^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	[= 1 – 0.004281 – 0.0005123 – 0.00002759]	**A1**	Correct unsimplified (or individual terms evaluated) answer seen Condone 1 – A + B + C leading to correct solution
	= 0.995	**B1**	B1 **not** dependent on previous marks.
	Alternative method for question 3(i)		
	P(at most 7) = P(0,1,2,3,4,5,6,7)	**M1**	Binomial term of form ${}^{10}C_xp^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	$= (0.65)^{10} + {}^{10}C1(0.35)^1(0.65)^9 + \ldots + {}^{10}C_7(0.35)^7(0.65)^3$	**A1**	Correct unsimplified answer or individual terms evaluated seen
	= 0.995	**B1**	
		3	
(ii)	$1 - (0.65)^n > 0.99$ $0.01 > (0.65)^n$	**M1**	Equation or inequality with $(0.65)^n$ **and** 0.01 or $(0.35)^n$ **and** 0.99 only (Note 1 – 0.99 is equivalent to 0.01 etc.)
	$n > 10.69$	**M1**	Solving their $a^n = c$, $0 < a,c < 1$ using logs **or** Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark
	smallest $n = 11$	**A1**	CAO
		3	

227. 9709_s19_qp_62 Q: 5

Maryam has 7 sweets in a tin; 6 are toffees and 1 is a chocolate. She chooses one sweet at random and takes it out. Her friend adds 3 chocolates to the tin. Then Maryam takes another sweet at random out of the tin.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Draw up the probability distribution table for the number of toffees taken. [3]

(iii) Find the mean number of toffees taken. [1]

....

....

....

....

....

(iv) Find the probability that the first sweet taken is a chocolate, given that the second sweet taken is a toffee. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

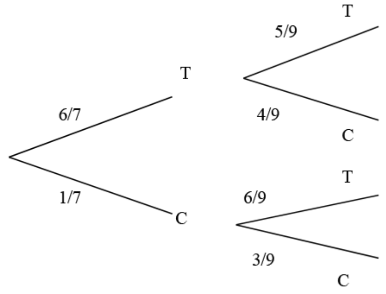
....

....

....

....

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td> </td><td>B1</td><td>First pair of branches labels and probs correct (6/7 and 1/7 or rounding to 0.857 and 0.143)
(Labelling must be logically…e.g. (T and T) or (T and Not T) would be acceptable)</td></tr>
<tr><td></td><td></td><td>B1</td><td>Either of second top pair or bottom of branches labels and probs correct</td></tr>
<tr><td></td><td></td><td>B1</td><td>Both second pairs of branches labels and probs correct. No additional / further branches.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td>No of toffees taken (T)</td><td>0</td><td>1</td><td>2</td></tr><tr><td>prob</td><td>$\frac{3}{63}$,
0.0476(2)</td><td>$\frac{30}{63}$,
0.476(2)</td><td>$\frac{30}{63}$,
0.476(2)</td></tr></table></td><td>B1</td><td>P(1) correct</td></tr>
<tr><td></td><td></td><td>B1</td><td>P(0) or P(2) correct</td></tr>
<tr><td></td><td></td><td>B1</td><td>FT Correct values in table, any additional values of T have stated probability of zero. For FT $\Sigma p = 1$,</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>(iii)</td><td>$E(X) = \frac{90}{63}$ $(\frac{10}{7})$ (1.43)</td><td>B1</td><td>Not FT</td></tr>
<tr><td></td><td></td><td>1</td><td></td></tr>
<tr><td>(iv)</td><td>$P(1^{st}\ C \mid 2^{nd}\ T) = \frac{P(C \cap T)}{P(T)} = \frac{\frac{1}{7}\times\frac{6}{9}}{\frac{1}{7}\times\frac{6}{9}+\frac{6}{7}\times\frac{5}{9}} = \frac{\frac{6}{63}}{\frac{36}{63}}$</td><td>B1</td><td>P(C∩T) attempt seen as numerator of a fraction, consistent with <i>their</i> tree diagram or correct</td></tr>
<tr><td></td><td></td><td>M1</td><td>Summing 2 appropriate two-factor probabilities, consistent with <i>their</i> tree diagram or correct seen anywhere</td></tr>
<tr><td></td><td></td><td>A1</td><td>$\frac{36}{63}$ oe or correct unsimplifed expression seen as numerator or denominator of a fraction</td></tr>
<tr><td></td><td>$\frac{1}{6}$ oe</td><td>A1</td><td>Final answer</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

228. 9709_s19_qp_63 Q: 6

A fair five-sided spinner has sides numbered 1, 1, 1, 2, 3. A fair three-sided spinner has sides numbered 1, 2, 3. Both spinners are spun once and the score is the product of the numbers on the sides the spinners land on.

(i) Draw up the probability distribution table for the score. [4]

(ii) Find the mean and the variance of the score. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(iii) Find the probability that the score is greater than the mean score. [2]

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$\begin{array}{\|l\|c\|c\|c\|c\|c\|c\|}\hline \text{score} & 1 & 2 & 3 & 4 & 6 & 9 \\ \hline \text{prob} & \frac{3}{15} & \frac{4}{15} & \frac{4}{15} & \frac{1}{15} & \frac{2}{15} & \frac{1}{15} \\ \hline \end{array}$	**B1**	Probability distribution table with correct scores, allow extra score values if probability of zero stated
		B1	2 probabilities (with correct score) correct
		B1	3 or more correct probabilities with correct scores
		B1	**FT** $\Sigma p = 1$, at least 4 probabilities
		4	
(ii)	mean = $\frac{(3+8+12+4+12+9)}{15} = \frac{48}{15}$ (3.2)	**B1**	
	Var = $\frac{(3+16+36+16+72+81)}{15} - (their\,3.2)^2$	**M1**	**FT** Substitute *their* attempts at scores in correct var formula, must have "– mean^2" (condone probabilities not summing to 1)
	$= \frac{224}{15} - 3.2^2 = 4.69 \left(\frac{352}{75}\right)$	**A1**	
		3	
(iii)	Score of 4, 6, 9	**M1**	Identifying relevant scores from *their* mean and *their* table
	Prob $\frac{4}{15}$ (0.267)	**A1**	Correct answer **SC** B1 for 4/15 with no working
		2	

229. 9709_w19_qp_61 Q: 2

Annan has designed a new logo for a sportswear company. A survey of a large number of customers found that 42% of customers rated the logo as good.

(i) A random sample of 10 customers is chosen. Find the probability that fewer than 8 of them rate the logo as good. [3]

....

....

....

....

....

....

....

....

....

....

....

(ii) On another occasion, a random sample of n customers of the company is chosen. Find the smallest value of n for which the probability that at least one person rates the logo as good is greater than 0.995. [3]

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$1-({}^{10}C_2\,0.42^8\,0.58^2+{}^{10}C_9\,0.42^9\,0.58^1+0.42^{10})$	**M1**	Binomial term of form ${}^{10}C_a p^a(1-p)^b$ $0<p<1$ any p, $0\leqslant a,b\leqslant 10$
		A1	Correct unsimplified expression
	0.983	**A1**	
		3	
(ii)	$1-P(0)>0.995$ $0.58^n<0.005$	**M1**	Equation or inequality involving 0.58^n or 0.42^n and 0.995 or 0.005
	$n>\dfrac{\log 0.005}{\log 0.58}$ $n>9.727$	**M1**	Attempt to solve using logs or Trial and Error. May be implied by their answer (rounded or truncated)
	$n=10$	**A1**	CAO
		3	

230. 9709_w19_qp_61 Q: 4

In a probability distribution the random variable X takes the values -1, 0, 1, 2, 4. The probability distribution table for X is as follows.

x	-1	0	1	2	4
$P(X = x)$	$\frac{1}{4}$	p	p	$\frac{3}{8}$	$4p$

(i) Find the value of p. [2]

(ii) Find $E(X)$ and $Var(X)$. [3]

....

....

....

....

....

....

....

(iii) Given that X is greater than zero, find the probability that X is equal to 2. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{1}{4}++p+p+\frac{3}{8}+4p=1$	**M1**	Unsimplified sum of probabilities equated to 1
	$p=\frac{1}{16}$	**A1**	If method FT from *their* incorrect **(i)**, expressions for E(*X*) and Var(*X*) must be seen unsimplified with all probabilities <1, condone not adding to 1
		2	
(ii)	$[\mathrm{E}(X)] = -\frac{1}{4}+\frac{1}{16}+\frac{6}{8}+1=\frac{25}{16}$	**M1**	May be implied by use in Variance, accept unsimplified
	$[\mathrm{Var}(X)] = \frac{1}{4}+\frac{1}{16}+\frac{12}{8}+\frac{16}{4}-\left(\mathit{their}\,\frac{25}{16}\right)^2$	**M1**	Substitute into correct variance formula, must have '– their mean^2'
	$\frac{863}{256}$ or 3.37	**A1**	OE
		3	
(iii)	$\mathrm{P}(X=2 \mid X>0) = \frac{\mathrm{P}(X=2)}{\mathrm{P}(X>0)} = \frac{\frac{3}{8}}{\frac{11}{16}}$	**M1**	Conditional probability formula used consistent with their probabilities
	$\frac{6}{11}$ or 0.545	**A1**	
		2	

231. 9709_w19_qp_62 Q: 5

A fair red spinner has four sides, numbered 1, 2, 3, 3. A fair blue spinner has three sides, numbered -1, 0, 2. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [4]

(ii) Find Var(X). [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td><table><tr><td>x</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>p</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td></tr></table></td><td>B1</td><td>Table with correct values of x, at least 1 probability, all probabilities $\leqslant 1$</td></tr>
<tr><td></td><td></td><td>B1</td><td>2 probabilities correct, may not be in table</td></tr>
<tr><td></td><td></td><td>B1</td><td>2 more probabilities correct, may not be in table</td></tr>
<tr><td></td><td></td><td>B1</td><td>All correct, values in table
SC1 No more than 1 correct probability and at least 5 probabilities summing to 1 in table</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td>(ii)</td><td>$[\mathrm{E}(X)] = \left(\frac{-1+0+3+4+9+8}{12}\right) = \frac{23}{12}$</td><td>M1</td><td>May be implied by use in variance. Allow unsimplified expression</td></tr>
<tr><td></td><td>$[\mathrm{Var}(X)] = \frac{1+0+3+8+27+32(=71)}{12} - \left(\frac{23}{12}\right)^2$</td><td>M1</td><td>Appropriate variance formula using <i>their</i> $\mathrm{E}(X)^2$</td></tr>
<tr><td></td><td>2.24 or $\frac{323}{144}$ or $2\frac{35}{144}$</td><td>A1</td><td>CAO</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

232. 9709_w19_qp_63 Q: 6

A box contains 3 red balls and 5 white balls. One ball is chosen at random from the box and is not returned to the box. A second ball is now chosen at random from the box.

(i) Find the probability that both balls chosen are red. [1]

....

....

....

....

(ii) Show that the probability that the balls chosen are of different colours is $\frac{15}{28}$. [2]

....

....

....

....

....

....

....

....

(iii) Given that the second ball chosen is red, find the probability that the first ball chosen is red. [2]

....

....

....

....

....

....

....

....

....

The random variable X denotes the number of red balls chosen.

(iv) Draw up the probability distribution table for X. [2]

(v) Find Var(X). [3]

Answer:

Question	Answer	Marks	Guidance
(i)	P(RR) = $\frac{3}{8}\times\frac{2}{7}=\frac{3}{28}$	**B1**	OE
		1	
(ii)	P(RW) + P(WR) $\frac{3}{8}\times\frac{5}{7}+\frac{5}{8}\times\frac{3}{7}$	**M1**	Method shown, numerical calculations identified, may include replacements
	$=\frac{15}{28}$	**A1**	AG, Fully correct calculations
	Alternative method for question 6(ii)		
	1 – (P(RR) + P(WW) $1-\left(\frac{3}{28}+\frac{5}{8}\times\frac{4}{7}\right)$	**M1**	Method shown, numerical calculations identified, may include replacements
	$=\frac{15}{28}$	**A1**	AG, Fully correct calculations
		2	
(iii)	P(first red|second red) = $\frac{\textit{their}\ \textbf{(i)}}{\textit{their}\ \textbf{(i)}+\frac{5}{8}\times\frac{3}{7}}=\frac{\frac{3}{8}\times\frac{2}{7}}{\frac{3}{8}\times\frac{2}{7}+\frac{5}{8}\times\frac{3}{7}}=\frac{\frac{3}{28}}{\frac{21}{56}}$	**M1**	Conditional probability formula used consistent with *their* probabilities or correct
	$=\frac{2}{7}$	**A1**	OE
		2	
Question	**Answer**	**Marks**	**Guidance**
(iv)	$\begin{array}{\|c\|c\|c\|c\|}\hline x & 0 & 1 & 2 \\ \hline p & \frac{10}{28} & \frac{15}{28} & \frac{3}{28} \\ \hline\end{array}$	**B1**	Probability distribution table with correct values of *x* and at least one correct probability placed. Extra *x* values allowed with probability of zero stated.
		B1FT	Fully correct FT P(2) = *their* **(i)**, P(1) = *their* **(ii)**, Σ(*p*) = 1.
		2	
(v)	$E(X)=\frac{30}{56}+\frac{12}{56}=\frac{42}{46}\quad\left(=\frac{3}{4}\right)$	**B1**	May be implied by use in variance formula
	$\mathrm{Var}(X)=\frac{30}{56}+\frac{24}{56}-\left(\textit{their}\ \frac{3}{4}\right)^2$	**M1**	Substitute into correct variance formula, must have '– *their* mean^2' Must be for 2 or more non-zero *x*-values
	$\frac{45}{112}$ or 0.402	**A1**	Correct final answer
		3	

233. 9709_m18_qp_62 Q: 4

The discrete random variable X has the following probability distribution.

x	-2	0	1	3	4
$\mathrm{P}(X = x)$	0.2	0.1	p	0.1	q

(i) Given that $\mathrm{E}(X) = 1.7$, find the values of p and q. [4]

(ii) Find $\mathrm{Var}(X)$. [2]

Answer:

Question	Answer	Marks	Guidance
(i)	$\Sigma p = 1$: $0.2 + 0.1 + p + 0.1 + q = 1$: $p + q = 0.6$	**M1**	Unsimplified sum of probabilities equated to 1
	$\Sigma px = 1.7$: $-0.4 + 0 + p + 0.3 + 4q = 1.7$:	**M1**	Unsimplified Sum of px equated to 1.7
	$p + 4q = 1.8$	**M1**	Solve simult. equations to find expression in p or q
	$p = 0.2$, $q = 0.4$	**A1**	
		4	
(ii)	$\text{Var}(X) = \Sigma px^2 - 1.7^2 = 4\text{x}0.2 + 1p + 9\text{x}0.1 + 16q - 1.7^2$ $= 8.3 - 2.89$	**M1**	Use correct unsimplified expression for variance
	$= 5.41$	**A1**	
		2	

234. 9709_s18_qp_61 Q: 3

Andy has 4 red socks and 8 black socks in his drawer. He takes 2 socks at random from his drawer.

(i) Find the probability that the socks taken are of different colours. [2]

The random variable X is the number of red socks taken.

(ii) Draw up the probability distribution table for X. [3]

(iii) Find $\mathrm{E}(X)$. [1]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>P(RB) + P(BR) = $\frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$ oe</td><td>M1</td><td>Multiply 2 probs together and summing two 2-factor probs, unsimplified, condone replacement</td></tr>
<tr><td></td><td>P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) (0.485) oe</td><td>A1</td><td>Correct answer</td></tr>
<tr><td></td><td>Method 2
1 – P(BB) – P(RR) = 1 – $\frac{4}{12} \times \frac{3}{11} - \frac{8}{12} \times \frac{7}{11}$</td><td>M1</td><td>Multiply 2 probs together and subtracting two 2-factor probs from 1, unsimplified, condone replacement</td></tr>
<tr><td></td><td>P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) oe</td><td>A1</td><td>Correct answer</td></tr>
<tr><td></td><td>Method 3
P(diff colours) = $\frac{({}^{4}C_{1} \times {}^{8}C_{1})}{{}^{12}C_{2}}$</td><td>M1</td><td>Multiply 2 combs together and dividing by a combination</td></tr>
<tr><td></td><td>= $\frac{16}{33}$</td><td>A1</td><td>Correct answer</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td>Number of red socks</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>$\frac{14}{33}$</td><td>$\frac{16}{33}$</td><td>$\frac{3}{33}$</td></tr></table></td><td>B1</td><td>Prob distribution table drawn, top row correct, condone additional values with $p = 0$ stated</td></tr>
<tr><td></td><td></td><td>B1</td><td>P(0) or P(2) correct to 3sf (need not be in table)</td></tr>
<tr><td></td><td></td><td>B1</td><td>All probs correct to 3sf, condone P(0) and P(2) swapped if correct</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
(iii)	E(X) = $1 \times \frac{16}{33} + 2 \times \frac{3}{33} = \frac{16}{33} + \frac{6}{33} = \frac{22}{33}$ ($\frac{2}{3}$)	B1ft	ft their table if 0, 1, 2 only, $0 < p < 1$
		1	

235. 9709_s18_qp_61 Q: 6

Vehicles approaching a certain road junction from town A can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town A, 55% turn left, 15% turn right and 30% go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.

(i) Find the probability that, of the next three vehicles approaching the junction from town A, one goes straight on and the other two either both turn left or both turn right. [4]

(ii) Three vehicles approach the junction from town A. Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on. [4]

Answer:

Question	Answer	Marks	Guidance
(i)	P(SLL) = (0.3)(0.55)(0.55) = 0.09075 ($\frac{363}{4000}$)	**M1**	P(SLL), P(SRR), P(SSL) or P(SSR) seen
	P(SRR) = (0.3)(0.15)(0.15) = 0.00675 ($\frac{27}{4000}$)	**A1**	Two correct options 0.09075 or 0.00675 can be unsimplified
	Total = ${}^3C_1 \times$ P(SLL) + ${}^3C_1 \times$ P(SRR) = 0.27225 + 0.02025	**M1**	Summing 6 prob options not all identical
	Prob = 0.293 accept 0.2925 ($\frac{117}{400}$)	**A1**	Correct answer
		4	
(ii)	$\text{P(SSS} \mid \text{all same dir}^n) = \frac{P(SSS\ and\ same\ dir^n)}{P(same\ direction)}$	**B1**	$(0.3)^3$ oe seen on its own as num or denom of a fraction
		M1	Attempt at P(*SSS*+*LLL*+*RRR*) seen anywhere
	$= \frac{0.3 \times 0.3 \times 0.3}{(0.15)^3 + (0.55)^3 + (0.3)^3}$	**A1**	$(0.15)^3 + (0.55)^3 + (0.3)^3$ oe seen as denom of a fraction
	= 0.137 ($\frac{108}{787}$)	**A1**	Correct answer
		4	

236. 9709_s18_qp_62 Q: 4

Mrs Rupal chooses 3 animals at random from 5 dogs and 2 cats. The random variable X is the number of cats chosen.

(i) Draw up the probability distribution table for X. [4]

(ii) You are given that $\mathrm{E}(X) = \frac{6}{7}$. Find the value of $\mathrm{Var}(X)$. [2]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>$\frac{2}{7}$</td><td>$\frac{4}{7}$</td><td>$\frac{1}{7}$</td></tr></table></td><td>B1</td><td>Prob distribution table drawn, top row correct with at least one probability $0 < p < 1$ entered, condone additional values with $p = 0$ stated</td></tr>
<tr><td></td><td>$P(0) = \frac{5}{7}\times\frac{4}{6}\times\frac{3}{5} = \frac{2}{7}$ (0.2857)</td><td>B1</td><td>One probability correct (need not be in table)</td></tr>
<tr><td></td><td>$P(1) = \frac{2}{7}\times\frac{5}{6}\times\frac{4}{5}\times {}^3C_1 = \frac{4}{7}$ (0.5713)</td><td>B1</td><td>Another probability correct (need not be in table).</td></tr>
<tr><td></td><td>$P(2) = \frac{2}{7}\times\frac{1}{6}\times\frac{5}{5}\times {}^3C_2 = \frac{1}{7}$ (0.1429)</td><td>B1</td><td>Values in table, all probs correct (to 3SF) or 3 probabilities summing to 1</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td>(ii)</td><td>$\text{Var}(X) = 1\times\frac{4}{7} + 4\times\frac{1}{7} - (\frac{6}{7})^2$
$= \frac{8}{7} - (\frac{6}{7})^2$</td><td>M1</td><td>Unsimplified correct numerical expression for variance
or <i>their</i> probabilities from (i) $0 < p < 1$ in unsimplified variance expression</td></tr>
<tr><td></td><td>$= \frac{20}{49}$ or 0.408</td><td>A1</td><td>Correct answer (0.40816…) nfww
Final answer does not imply the method mark</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
</table>

237. 9709_s18_qp_63 Q: 2

The random variable X has the distribution $\mathrm{N}(-3, \sigma^2)$. The probability that a randomly chosen value of X is positive is 0.25.

(i) Find the value of σ. [3]

....

....

....

....

....

....

....

....

....

....

....

(ii) Find the probability that, of 8 random values of X, fewer than 2 will be positive. [3]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$z = 0.674$	**B1**	z value ±0.674
	$0.674 = \frac{0 - -3}{\sigma}$	**M1**	±Standardising with 0 and equating to a z-value
	σ = 4.45	**A1**	Correct answer www ie not ignoring a minus sign
	Total:	**3**	
(ii)	P(0, 1)	**M1**	Any bin of form ${}^8C_x(0.75)^x\ (0.25)^{8-x}$any x
	$= (0.75)^8 + {}^8C_1(0.25)(0.75)^7$	**M1**	Correct unsimplified answer, may be implied by numerical values
	0.1001+ 0.2670 = 0.367	**A1**	Correct answer
	Method 2 $1 - P(8,7,6,5,4,3,2) = 1 - (0.25)^8 - {}^8C_1(0.75)(0.25)^7 - \ldots$	**M1**	Any bin of form ${}^8C_x(0.75)^x\ (0.25)^{8-x}$any x
	$- {}^8C_2(0.75)^6\ (0.25)^2$	**M1**	Correct unsimplified answer
	= 0.367	**A1**	Correct answer
	Total:	**3**	

238. 9709_s18_qp_63 Q: 5

A game is played with 3 coins, A, B and C. Coins A and B are biased so that the probability of obtaining a head is 0.4 for coin A and 0.75 for coin B. Coin C is not biased. The 3 coins are thrown once.

(i) Draw up the probability distribution table for the number of heads obtained. [5]

(ii) Hence calculate the mean and variance of the number of heads obtained. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="6">(i)</td><td rowspan="2">P(0) = 0.6 × 0.25 × 0.5 = 0.075
P(1) = 0.4 × 0.25 × 0.5 + 0.6 × 0.75 × 0.5 + 0.6 × 0.25 × 0.5 = 0.35
P(2) = 0.4 × 0.75 × 0.5 + 0.4 × 0.25 × 0.5 + 0.6 × 0.75 × 0.5 = 0.425
P(3) = 0.4 × 0.75 × 0.5 = 0.15</td><td>B1</td><td>0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)</td></tr>
<tr><td>M1</td><td>Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table</td></tr>
<tr><td rowspan="3"><table><tr><td>No of heads</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>0.075 $\left(\frac{3}{40}\right)$</td><td>0.35 $\left(\frac{7}{20}\right)$</td><td>0.425 $\left(\frac{17}{40}\right)$</td><td>0.15 $\left(\frac{3}{20}\right)$</td></tr></table></td><td>M1</td><td>Summing 3 probabilities for P(1) or P(2) with or without a table</td></tr>
<tr><td>B1</td><td>One correct probability seen.</td></tr>
<tr><td>A1</td><td>All correct in a table</td></tr>
<tr><td>Total:</td><td>5</td><td></td></tr>
<tr><td>(ii)</td><td>E(<i>X</i>) = 0.35 + 2 × 0.425 + 3 × 0.15 = 1.65 $\left(\frac{33}{20}\text{ oe}\right)$</td><td>M1</td><td>Correct unsimplified expression for the mean using their table, ∑p = 1; can be implied by correct answer</td></tr>
<tr><td rowspan="3">(ii)</td><td>Var(<i>X</i>) = 0.35 + 4 × 0.425 + 9 × 0.15 – 1.65^2</td><td>M1</td><td>Correct unsimplified expression for the variance using their table and their mean^2 subtracted, ∑p = 1</td></tr>
<tr><td>= 0.678 (0.6775) $\left(\frac{271}{400}\text{ oe}\right)$</td><td>A1</td><td>Correct answer</td></tr>
<tr><td>Total:</td><td>3</td><td></td></tr>
</table>

239. 9709_w18_qp_61 Q: 2

A random variable X has the probability distribution shown in the following table, where p is a constant.

x	-1	0	1	2	4
$\mathrm{P}(X = x)$	p	p	$2p$	$2p$	0.1

(i) Find the value of p. [1]

....

....

....

....

(ii) Given that $\mathrm{E}(X) = 1.15$, find $\mathrm{Var}(X)$. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$6p + 0.1 = 1$ $p = 0.15$	**B1**	Correct answer
		1	
(ii)	$\text{Var}(X) = 1 \times p + 1 \times 2p + 4\times 2p + 16 \times 0.1 - 1.15^2$	**M1**	Correct unsimplified formula, *their p* substituted (allow 1 error)
	$0.15 + 0 + 0.3 + 1.2 + 1.6 - 1.15^2$ $= 1.9275 = 1.93$ (3sf)	**A1**	Correct answer
		2	

240. 9709_w18_qp_62 Q: 3

Jake attempts the crossword puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75, independently of all other days.

(i) Find the probability that he will complete the puzzle at least three times over a period of five days. [3]

Kenny also attempts the puzzle every day. The probability that he will complete the puzzle on a Monday is 0.8. The probability that he will complete it on a Tuesday is 0.9 if he completed it on the previous day and 0.6 if he did not complete it on the previous day.

(ii) Find the probability that Kenny will complete the puzzle on at least one of the two days Monday and Tuesday in a randomly chosen week. [3]

Answer:

Question	Answer	Marks	Guidance
(i)	**Method 1**		
	P(3) + P(4) + P(5) = $^5C_3\ 0.75^3 \times 0.25^2$ +	**M1**	One binomial term $^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	$^5C_4\ 0.75^4 \times 0.25^1 + {}^5C_5\ 0.75^5 \times 0.25^0$	**M1**	Correct unsimplified expression
	= 0.26367 + 0.39551 + 0.23730 = 0.896 (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
	Method 2		
	1 – P(0) – P(1) – P(2) = $1 - {}^5C_0\ 0.75^0 \times 0.25^5$	**M1**	One binomial term $^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	$- {}^5C_1\ 0.75^1 \times 0.25^4 - {}^5C_2\ 0.75^2 \times 0.25^3$	**M1**	Correct simplified expression
	= 1 – 0.00097656 – 0.014648 – 0.087891 = 0.896 (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
		3	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	P(C,C) + P(C,C′) + P(C′,C) 0.8 × 0.9	**B1**	Unsimplified prob completed on both days
	0.8 × 0.1 + 0.2 × 0.6	**M1**	Unsimplified prob 0.8 × a + 0.2 × b, a = 0.1or 0.4, b = 0.6 or 0.9
	= 0.92 oe	**A1**	Correct final answer
	Method 2		
	1 – P(C′,C′) = 1 – 0.2 × 0.4	**B1**	Unsimplified prob completed on no days
		M1	1 – 0.2 × a, a=0.1or 0.4 allow unsimplified
	= 0.92	**A1**	Correct final answer
		3	

$$P(S,S') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$$

$$P(P,P') = \frac{2}{11} \times \frac{9}{10} = \frac{18}{110}$$

$$P(I,I') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$$

$$P(M,M') = \frac{1}{11} \times \frac{10}{10} = \frac{10}{110}$$

$$Total = \frac{84}{110}$$

$$P(Same) = 1 - \frac{84}{110} = \frac{26}{110}$$

B1 one of products correct
M1 1 – sum of probabilities from 4 appropriate scenarios
A1 Correct final answer

---

241. 9709_w18_qp_62 Q: 6

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 3 sides, numbered 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [3]

(ii) Find Var(X). [3]

....

....

....

....

....

....

....

....

....

....

....

....

(iii) Find the probability that X is equal to 1, given that X is non-zero. [3]

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$\begin{array}{ccccccc} x & -2 & -1 & 0 & 1 & 2 & 3 \\ p & \frac{1}{12} & \frac{2}{12} & \frac{3}{12} & \frac{3}{12} & \frac{2}{12} & \frac{1}{12} \end{array}$	**B1**	–2, –1, 0, 1, 2, 3 seen as top line of a pdf table with at least 1 probability OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3) (condone additional values with *p*=0 stated)
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	
(ii)	E(X) = $\frac{-2\times1-1\times2+0+1\times3+2\times2+1\times3}{12}=0.5$	**M1**	Unsimplified expression for mean using *their* pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.
	Var(X) = $\frac{(-2)^2\times1+(-1)^2\times2+1^2\times3+2^2\times2+3^2\times1}{12}-(\mathit{their}\,0.5)^2$	**M1**	Unsimplified expression for variance using *their* pdf table (or correct) with at least 2 non-zero values and *their* E(X). Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly
	26/12 −1/4 = 23/12	**A1**	Correct final answer
		3	
(iii)	**Method 1**		
	P(X non-zero) = 9/12	**B1ft**	If Binomial distribution used 0/3 P(X non-zero) ft from *their* pdf table, Σ*p*=1 oe
	P(X = 1 | X non-zero) = $\frac{P(X=1\cap X\ non-zero)}{P(X\ non-zero)}=\frac{3/12}{9/12}$	**M1**	*Their* P(X = 1)/*their* P(X non-zero) from *their* pdf table oe
	= 1/3 oe	**A1**	Correct final answer www
	Method 2		
	P(X = 1 | X non-zero) = $\frac{Number\ of\ outcomes=1}{Number\ of\ non-zero\ outcomes}$	**B1ft**	Number of non-zero outcomes (expect 9) ft from *their* outcome table or pdf table numerators oe
		M1	*a*/*b*, *a* = *their* 3 from *their* outcome table or pdf table numerators, *b* = *their* 9 (not 12)
	$=\frac{3}{9}=\frac{1}{3}$ oe	**A1**	Correct final answer www
		3	

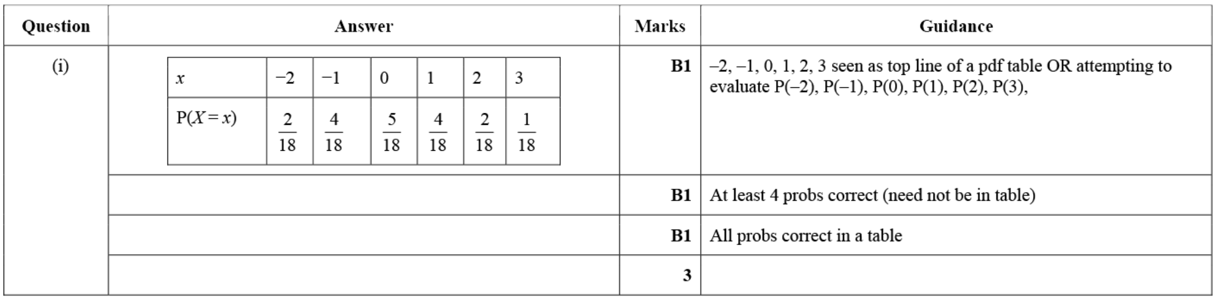
242. 9709_w18_qp_63 Q: 2

A fair 6-sided die has the numbers -1, -1, 0, 0, 1, 2 on its faces. A fair 3-sided spinner has edges numbered -1, 0, 1. The die is thrown and the spinner is spun. The number on the uppermost face of the die and the number on the edge on which the spinner comes to rest are noted. The sum of these two numbers is denoted by X.

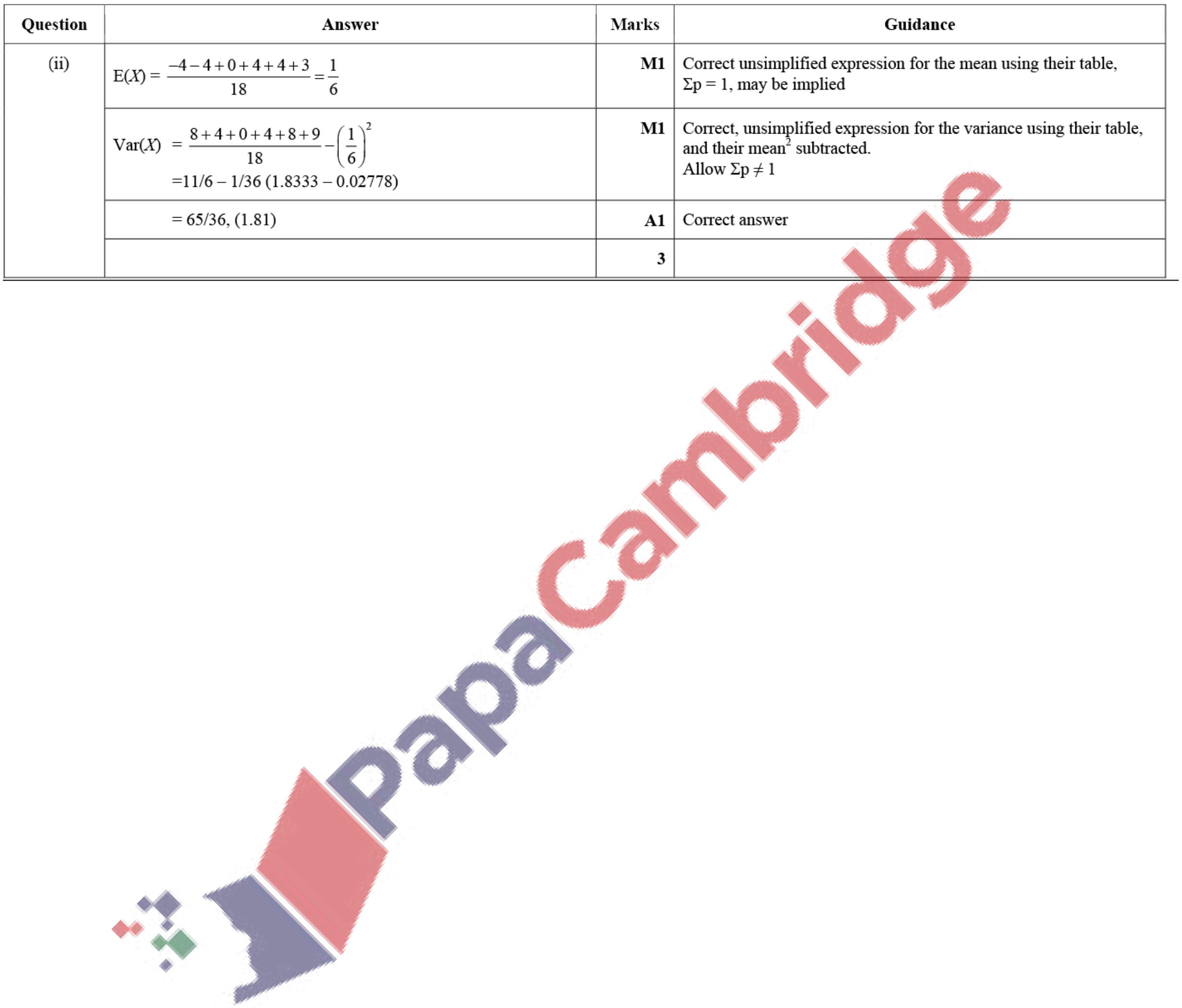
(i) Draw up a table showing the probability distribution of X. [3]

(ii) Find Var(X). [3]

Answer:



<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td><table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X=x)$</td><td>$\frac{2}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{5}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{2}{18}$</td><td>$\frac{1}{18}$</td></tr></table></td><td>B1</td><td>–2, –1, 0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3),</td></tr>
<tr><td></td><td></td><td>B1</td><td>At least 4 probs correct (need not be in table)</td></tr>
<tr><td></td><td></td><td>B1</td><td>All probs correct in a table</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>



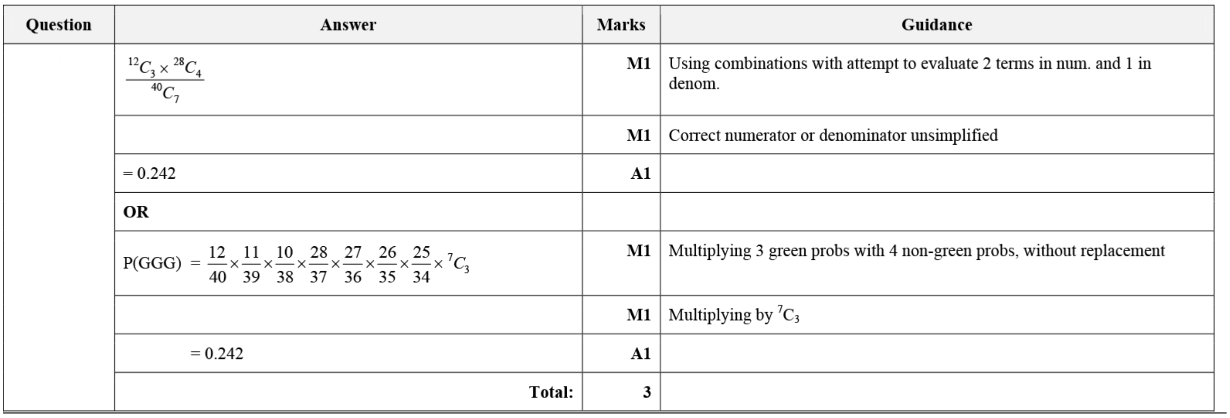
Question	Answer	Marks	Guidance
(ii)	$E(X)=\frac{-4-4+0+4+4+3}{18}=\frac{1}{6}$	M1	Correct unsimplified expression for the mean using their table, Σp = 1, may be implied
	$\text{Var}(X)=\frac{8+4+0+4+8+9}{18}-\left(\frac{1}{6}\right)^2$ =11/6 – 1/36 (1.8333 – 0.02778)	M1	Correct, unsimplified expression for the variance using their table, and their mean^2 subtracted. Allow Σp ≠ 1
	= 65/36, (1.81)	A1	Correct answer
		3	

243. 9709_m17_qp_62 Q: 2

A bag contains 10 pink balloons, 9 yellow balloons, 12 green balloons and 9 white balloons. 7 balloons are selected at random without replacement. Find the probability that exactly 3 of them are green.

[3]

Answer:



Question	Answer	Marks	Guidance
	$\frac{{}^{12}C_3 \times {}^{28}C_4}{{}^{40}C_7}$	**M1**	Using combinations with attempt to evaluate 2 terms in num. and 1 in denom.
		M1	Correct numerator or denominator unsimplified
	= 0.242	**A1**	
	OR		
	P(GGG) $= \frac{12}{40}\times\frac{11}{39}\times\frac{10}{38}\times\frac{28}{37}\times\frac{27}{36}\times\frac{26}{35}\times\frac{25}{34}\times{}^{7}C_3$	**M1**	Multiplying 3 green probs with 4 non-green probs, without replacement
		M1	Multiplying by ${}^{7}C_3$
	= 0.242	**A1**	
	Total:	**3**	

244. 9709_m17_qp_62 Q: 6

Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(i) Show that $\mathrm{P}(X = 2) = \frac{2}{15}$. [2]

(ii) Draw up the probability distribution table for X. [4]

(iii) Given that $X = 3$, find the probability that the card chosen from pack A is a 1. [3]

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td>P(2) = P(0,2) = $2/10 \times 4/6$</td><td>M1</td><td>Mult 2 probs seen (or complete listing of all options)</td></tr>
<tr><td></td><td>= 2/15 AG</td><td>A1</td><td>Correct answer legit obtained</td></tr>
<tr><td></td><td>Total:</td><td>2</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>5</td></tr><tr><td>$P(X=x)$</td><td>2/30</td><td>5/30</td><td>4/30</td><td>13/30</td><td>6/30</td></tr></table></td><td>B1</td><td>Correct values for x in table. Any additional values must have $P(x)=0$ stated</td></tr>
<tr><td></td><td></td><td>B1</td><td>One correct prob other than P(2) or P(3)</td></tr>
<tr><td></td><td></td><td>B1</td><td>Correct P(3)</td></tr>
<tr><td></td><td></td><td>B1</td><td>All correct</td></tr>
<tr><td></td><td>Total:</td><td>4</td><td></td></tr>
<tr><td>(iii)</td><td>$\mathrm{P}(A1 \mid \text{Sum } 3) = \dfrac{\mathrm{P}(A1 \cap \text{Sum3})}{\mathrm{P}(\text{Sum3})} = \dfrac{5/10 \times 4/6}{13/30}$</td><td>M1</td><td>Attempt at P($A1 \cap$ Sum 3) as num or denom of a fraction, can be by counting</td></tr>
<tr><td></td><td></td><td>M1</td><td>Their P(3) from (ii) as num or denom of a fraction</td></tr>
<tr><td></td><td>= 10/13(0.769)</td><td>A1</td><td></td></tr>
<tr><td></td><td>Total:</td><td>3</td><td></td></tr>
</table>

245. 9709_s17_qp_61 Q: 5

Eggs are sold in boxes of 20. Cracked eggs occur independently and the mean number of cracked eggs in a box is 1.4.

(i) Calculate the probability that a randomly chosen box contains exactly 2 cracked eggs. [3]

(ii) Calculate the probability that a randomly chosen box contains at least 1 cracked egg. [2]

....

....

....

....

....

....

....

....

....

....

....

iii) A shop sells *n* of these boxes of eggs. Find the smallest value of *n* such that the probability of there being at least 1 cracked egg in each box sold is less than 0.01. [2]

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

(i)	$p = 0.07$	**B1**	
	$\mathrm{P}(2) = {}^{20}\mathrm{C}_2 (0.07)^2 (0.93)^{18}$	**M1**	Bin term ${}^{20}\mathrm{C}_x p^x (1-p)^{20-x}$ their p
	$= 0.252$	**A1**	
	Total:	**3**	
(ii)	P(at least 1 cracked egg)=$1-(0.93)^{20}=1-0.2342$	**M1**	Attempt to find P(at least1 cracked egg) with their p from (i) allow 1 – P(0, 1) OE
	$= 0.766$	**A1**	Rounding to 0.766
	Total:	**2**	
(iii)	$(0.7658)^n<0.01$	**M1**	Eqn or inequal containing (their $0.766)^n$ or (their $0.234)^n$, together with 0.01 or 0.99
	$n = 18$	**A1**	
	Total:	**2**	

246. 9709_s17_qp_62 Q: 3

In a probability distribution the random variable X takes the value x with probability kx^2, where k is a constant and x takes values -2, -1, 2, 4 only.

(i) Show that $P(X = -2)$ has the same value as $P(X = 2)$. [1]

....

....

....

....

(ii) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

....

....

....

....

....

....

....

....

....

....

(iii) Find $E(X)$. [2]

....

....

....

....

....

....

....

Answer:

<table>
<tr><td>(i)</td><td>$k(-2)^2$ is the same as $k(2)^2 = 4k$</td><td>B1</td><td>need to see $-2^2\,k$, 2^2k and $4k$, algebraically correct expressions OE</td></tr>
<tr><td></td><td>Total:</td><td>1</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td>x</td><td>–2</td><td>–1</td><td>2</td><td>4</td></tr><tr><td>Prob</td><td>$4k$</td><td>k</td><td>$4k$</td><td>$16k$</td></tr></table></td><td>B1</td><td>–2, -1, 2, 4 only seen in a table, together with at least one attempted probability involving k</td></tr>
<tr><td></td><td>$4k + k + 4k + 16k = 1$</td><td>M1</td><td>Summing 4 probs equating to 1. Must all be positive (table not required)</td></tr>
<tr><td></td><td>$k = 1/25$ (0.04)</td><td>A1</td><td>CWO</td></tr>
<tr><td></td><td>Total:</td><td>3</td><td></td></tr>
<tr><td>(iii)</td><td>$\mathrm{E}(X) = -8k + -k + 8k + 64k = 63k$</td><td>M1</td><td>using Σpx unsimplified. FT their k substituted before this stage, no inappropriate dividing</td></tr>
<tr><td></td><td>$= 63/25$ (2.52)</td><td>A1</td><td></td></tr>
<tr><td></td><td>Total:</td><td>2</td><td></td></tr>
</table>

247. 9709_s17_qp_62 Q: 4

Two identical biased triangular spinners with sides marked 1, 2 and 3 are spun. For each spinner, the probabilities of landing on the sides marked 1, 2 and 3 are p, q and r respectively. The score is the sum of the numbers on the sides on which the spinners land. You are given that P(score is 6) = $\frac{1}{36}$ and P(score is 5) = $\frac{1}{9}$. Find the values of p, q and r. [6]

Answer:

Question	Answer	Marks	Guidance
	P(score is 6) = P(3, 3)	**M1**	Realising that score 6 is only P(3, 3)
	$= r^2 = 1/36$ $r = 1/6$	**A1**	Correct ans [SR **B2** $r = 1/6$ without workings]
	P(2, 3) + P(3, 2) = 1/9 $qr + rq = 1/9$	**M1**	Eqn involving qr (OE) equated to 1/9 (r may be replaced by *their* ‘r value’)
	$q/6 + q/6 = 1/9$	**M1**	Correct equation with *their* ‘r value’ substituted
	$q = 1/3$	**A1**	Correct answer seen, does **not** imply previous M’s
	$p = 1 – 1/6 – 1/3 = 1/2$	**B1 FT**	FT their p + their r + their q =1 , $0 < p < 1$
	Total:	**6**	

248. 9709_s17_qp_62 Q: 7

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

(i) Find the probability that Khalid hurts himself on any particular day. [2]

(ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]

(iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]

....

....

....

....

....

....

....

....

....

....

....

(iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	P(H) = P(BH) + P(SH) = 0.6×0.05 + 0.4×0.75	**M1**	Summing two 2-factor probs using 0.6 with 0.05 or 0.95, and 0.4 with 0.75 or 0.25
	$= 0.330$ or $\frac{33}{100}$	**A1**	Correct final answer accept 0.33
	Total:	**2**	
(ii)	$\text{P}\left(S\middle\|H\right) = \frac{P(S\cap H)}{P(H)} = \frac{0.4\times0.75}{0.33} = \frac{0.3}{0.33}$	**M1 FT**	Their $\frac{P(S\cap H)}{P(H)}$ unsimplified, FT from (i)
	$= \frac{10}{11}$ or 0.909	**A1**	
	Total:	**2**	
(iii)	Var (B) = 45×0.6×0.4 Var (S)= 45×0.4×0.6	**B1**	One variance stated unsimplified
	Variances same	**B1**	Second variance stated unsimplified **and** at least one variance clearly identified, **and** both evaluated *or* showing equal *or* conclusion made SR **B1** – Standard Deviation calculated Fulfil all the criteria for the variance method but calculated to Standard Deviation
	Total:	**2**	
(iv)	1 – P(0, 1) $= 1 - [(0.6)^{10} + {}^{10}C_1(0.4)(0.6)^9] = 1 - 0.0464$ OR P(2,3,4,5,6,7,8,9,10) $= {}^{10}C_2(0.4)^2(0.6)^8 + \ldots + {}^{10}C_9(0.4)^9(0.6) + (0.4)^{10}$	**M1** **M1**	Bin term ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ Correct unsimplified answer
	= 0.954	**A1**	
	Total:	**3**	

249. 9709_s17_qp_63 Q: 5

Hebe attempts a crossword puzzle every day. The number of puzzles she completes in a week (7 days) is denoted by X.

(i) State two conditions that are required for X to have a binomial distribution. [2]

On average, Hebe completes 7 out of 10 of these puzzles.

(ii) Use a binomial distribution to find the probability that Hebe completes at least 5 puzzles in a week. [3]

(iii) Use a binomial distribution to find the probability that, over the next 10 weeks, Hebe completes 4 or fewer puzzles in exactly 3 of the 10 weeks. [3]

Answer:

Question	Answer	Marks	Guidance
(i)	constant probability (of completing)	**B1**	Any one condition of these two
	independent trials/events	**B1**	The other condition
	Totals:	**2**	
(ii)	P(5, 6, 7) = $^7C_5(0.7)^5(0.3)^2 + {}^7C_6(0.7)^6(0.3)^1 + (0.7)^7$	**M1** **A1**	Bin term $^7C_x(0.7)^x(0.3)^{7-x}$, $x \neq 0, 7$ Correct unsimplified answer (sum) OE
	= 0.647	**A1**	
	Total:	**3**	
(iii)	P(0, 1, 2, 3, 4) = 1 – their '0.6471' = 0.3529	**M1**	Find P($\leqslant 4$) either by subtracting their (ii) from 1 or from adding Probs of 0,1,2,3,4 with n=7 (or 10) and p = 0.7
	P(3) = $^{10}C_3(0.3529)^3(0.6471)^7$	**M1**	$^{10}C_3$ (their 0.353)^3(1 – their 0.353)^7 on its own
	= 0.251	**A1**	

250. 9709_s17_qp_63 Q: 6

Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,

(i) if digits are not repeated, [2]

(ii) if digits can be repeated and the number formed is odd. [3]

(b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.

(i) Find the number of different selections that include the 2 oatmeal biscuits. [2]

(ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

Answer:

(a)(i)	First digit in 2 ways. 2 × 4 × 3 × 2 or 2 × 4P3	**M1**	1, 2 or 3 × 4P3 OE as final answer
	Total = 48 ways	**A1**	
	Total:	**2**	
(a)(ii)	2 × 5 × 5 × 3	**M1** **M1**	Seeing 5^2 mult; this mark is for correctly considering the middle two digits with replacement Mult by 6; this mark is for correctly considering the first and last digits
	= 150 ways	**A1**	
	Totals:	**3**	

Question	Answer	Marks	Guidance
(b)(i)	OO**** in $^{18}C_4$ ways	**M1**	$^{18}C_x$ or the sum of five 2-factor products with n = 14 and 4, may be × by 2C2: 4C0 × 14C4 + 4C1 × 14C3 + 4C2 × 14C2 + 4C3 × 14C1 + 4C4 (× 14C0)
	= 3060	**A1**	
	Totals:	**2**	

Question	Answer	Marks	Guidance
(b)(ii)	Choc Not Choc 0 6= 1 × $^{16}C_6$ = 8008 0.2066 1 5= $^4C_1$ × $^{16}C_5$ = 17472 0.4508 2 4= $^4C_2$ × $^{16}C_4$ = 10920 0.2817 OR Choc Oats Ginger 0 0 6 0 1 5 0 2 4 1 0 5 1 1 4 1 2 3 2 0 4 2 1 3 2 2 2	**B1**	The correct number of ways with one of 0, 1 or 2 chocs , unsimplified **or** any three correct number of ways of combining choc/oat/ginger, unsimplified
	Total = 36400 ways	**M1**	sum the number of ways with 0, 1 and 2 chocs and two must be totally correct, unsimplified OR sum the nine combinations of choc, ginger, oats, six must be totally correct, unsimplified
	Probability = 36400/ $^{20}C_6$	**M1**	dividing by $^{20}C_6$ (38760) oe
	= 0.939 (910/969)	**A1**	
	Totals:	**4**	

251. 9709_w17_qp_61 Q: 1

The discrete random variable X has the following probability distribution.

x	1	2	3	6
$P(X = x)$	0.15	p	0.4	q

Given that $E(X) = 3.05$, find the values of p and q. [4]

Answer:

Question	Answer	Marks	Guidance
	$p + q = 0.45$	**M1**	Equation involving $\Sigma P(x) = 1$
	$0.15 + 2p + 1.2 + 6q = 3.05$	**M1**	Equation using $E(X) = 3.05$
	$q = 0.2$	**M1**	Solving simultaneous equations to one variable
	$p = 0.25$	**A1**	Both answers correct
		4	

252. 9709_w17_qp_61 Q: 3

An experiment consists of throwing a biased die 30 times and noting the number of 4s obtained. This experiment was repeated many times and the average number of 4s obtained in 30 throws was found to be 6.21.

(i) Estimate the probability of throwing a 4. [1]

....

....

....

Hence

(ii) find the variance of the number of 4s obtained in 30 throws, [1]

....

....

....

(iii) find the probability that in 15 throws the number of 4s obtained is 2 or more. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	p = 0.207	**B1**	
		1	
(ii)	Var = 30 × 0.207 × 0.793 = 4.92	**B1**	
		1	
(iii)	P(⩾ 2) = 1 – P(0, 1)	**M1**	
	= 1 – (0.793)15 – $\binom{15}{1}$(0.207)(0.793)14	**M1**	1 – P(0, 1) seen n =15 p = any prob
	= 0.848	**A1**	
		3	

253. 9709_w17_qp_62 Q: 3

A box contains 6 identical-sized discs, of which 4 are blue and 2 are red. Discs are taken at random from the box in turn and not replaced. Let X be the number of discs taken, up to and including the first blue one.

(i) Show that $\mathrm{P}(X = 3) = \frac{1}{15}$. [2]

(ii) Draw up the probability distribution table for X. [3]

Answer:

Question	Answer	Marks	Guidance
(i)	*EITHER:* $P(X=3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	**(M1**	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$, accept × 1 as $\frac{4}{r}$.
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR1: $P(X=3) = P(RRB) = \frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or × 1 or $\times \frac{4}{4}$ included
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR2: $P(X=3) = P(RRB) = \frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities in order $\frac{{}^2C_1}{{}^pC_1} \times \frac{{}^1C_1}{{}^qC_1} \times \frac{{}^4C_1}{{}^rC_1}$ $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or × 1 or $\times \frac{4}{4}$ acceptable)
	= 1/15 AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
		2	

Question	Answer	Marks	Guidance
(ii)	$P(1) = P(B) = \frac{4}{6}$ $(\frac{2}{3} = 0.667)$	**B1**	Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leqslant p < 1$.
	$P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ $(= 0.267)$	**B1**	P(1) or P(2) correct unsimplified, or better, and identified.
	$P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15}$ $(= 0.0667)$ $\begin{array}{\|l\|l\|l\|l\|}\hline x & 1 & 2 & 3 \\ \hline P & \frac{10}{15} & \frac{4}{15} & \frac{1}{15} \\ \hline\end{array}$	**B1**	All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0
		3	

254. 9709_w17_qp_62 Q: 4

A fair tetrahedral die has faces numbered 1, 2, 3, 4. A coin is biased so that the probability of showing a head when thrown is $\frac{1}{3}$. The die is thrown once and the number n that it lands on is noted. The biased coin is then thrown n times. So, for example, if the die lands on 3, the coin is thrown 3 times.

(i) Find the probability that the die lands on 4 and the number of times the coin shows heads is 2. [3]

....

....

....

....

(ii) Find the probability that the die lands on 3 and the number of times the coin shows heads is 3. [1]

....

....

....

....

(iii) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. [3]

....

....

....

....

....

....

....

....

....

....

....

Answer:

Question	Answer	Marks	Guidance
(i)	$P(4, 2H) = \frac{1}{4} \times {}^4C_2 \times (\frac{1}{3})^2 (\frac{2}{3})^2$	M1	Multiplying their 2H expression by ¼ [P(4)]
		M1	Remaining factor is $(\frac{1}{3})^2(\frac{2}{3})^2$ [or $\frac{4}{81}$] multiplied by integer value $k \geqslant 1$ OE
	$= \frac{2}{27}$ (0.0741)	A1	
		3	
(ii)	$P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.00926)	B1	
		1	
(iii)	$P(1, 1H) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (0.08333) $P(2, 2H) = \frac{1}{4} \times (\frac{1}{3})^2 = \frac{1}{36}$ (0.02778) $P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.009259) $P(4, 4H) = \frac{1}{4} \times (\frac{1}{3})^4 = \frac{1}{324}$ (0.003086)	M1	Correct expression for 1 of P(1, 1H), P(2, 2H), P(4, 4H) Unsimplified (or better)
		M1	Summing their values for 3 or 4 appropriate outcomes for the ‘game’ with no additional outcomes.
	Prob $= \frac{10}{81}$ (0.123)	A1	
		3	

255. 9709_w17_qp_63 Q: 1

A statistics student asks people to complete a survey. The probability that a randomly chosen person agrees to complete the survey is 0.2. Find the probability that at least one of the first three people asked agrees to complete the survey. [2]

Answer:

Question	Answer	Marks	Guidance
	EITHER: P(at least 1 completes) = 1 – P(0 people complete) $= 1 - (0.8)^3$	(M1	Fully correct unsimplified expression $1 - (0.8)^3$ OE
	$= 0.488\ \left(\frac{61}{125}\right)$	A1)	
	OR1: $P(1, 2, 3) = {}^3C_1(0.2)(0.8)^2 + {}^3C_2(0.2)^2(0.8) + (0.2)^3$	(M1	Unsimplified correct 3 term expression
	$= 0.488\ \left(\frac{61}{125}\right)$	A1)	
	OR2: $0.2 + 0.8 \times 0.2 + 0.8 \times 0.8 \times 0.2$	(M1	Unsimplified sum of 3 correct terms
	$= 0.488\ \left(\frac{61}{125}\right)$	A1)	
		2	

256. 9709_w17_qp_63 Q: 4

A fair die with faces numbered 1, 2, 2, 2, 3, 6 is thrown. The score, X, is found by squaring the number on the face the die shows and then subtracting 4.

(i) Draw up a table to show the probability distribution of X. [3]

....

....

....

....

....

....

....

....

....

....

....

(ii) Find $E(X)$ and $Var(X)$. [3]

....

....

....

....

....

....

....

....

....

....

....

Answer:

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>(i)</td><td rowspan="3"><table><tr><td>x</td><td>–3</td><td>0</td><td>5</td><td>32</td></tr><tr><td>Prob</td><td>1/6</td><td>1/2</td><td>1/6</td><td>1/6</td></tr></table></td><td>B1</td><td>At least 3 different correct values of X (can be unsimplified)</td></tr>
<tr><td></td><td>B1</td><td>Four correct probabilities in a Probability Distribution table</td></tr>
<tr><td></td><td>B1</td><td>Correct probs with correct values of X</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
(ii)	$E(X) = –3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	M1	Subst their attempts at scores in correct formula as long as 'probs' sum to 1
	$\text{Var}(X) = 9/6 + 25/6 + 1024/6 – (34/6)^2$	M1	Subst their attempts at scores in correct var formula
	$= 144 \left(\frac{1298}{9}\right)$	A1	Both answers correct
		3	

257. 9709_m16_qp_62 Q: 2

A flower shop has 5 yellow roses, 3 red roses and 2 white roses. Martin chooses 3 roses at random. Draw up the probability distribution table for the number of white roses Martin chooses. [4]

Answer:

<table>
<tr><td></td><td><table><tr><td>No of W</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>42/90</td><td>42/90</td><td>6/90</td></tr></table></td><td>B1</td><td>0, 1, 2, seen in table with attempt at prob.</td></tr>
<tr><td></td><td>P(0) = 8/10 × 7/9 × 6/8 = 42/90</td><td>M1</td><td>3-factor prob seen with different denoms.</td></tr>
<tr><td></td><td>P(1W) = P(W,NW, NW) × 3 = 2/10 × 8/9 × 7/8 × 3
= 42/90</td><td>M1</td><td>Mult by 3</td></tr>
<tr><td></td><td>P(2W) = P(W, W, NW) × 3 = 2/10 × 1/9 × 8/8 × 3
= 6/90</td><td>A1 4</td><td>All correct</td></tr>
</table>

258. 9709_s16_qp_61 Q: 2

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

Answer:

	P (throwing a 4) = (1 – 0.4) / 4 = 0.15	**M1** **A1**	Sensible attempt to find P(1) Correct answer
	P(at most 1) = P(0, 1) or 1 – P(2, 3) = $(0.85)^3 + {}^3C_1\ (0.15)\ (0.85)^2$	**M1** **M1**	A binomial term with ${}^3C_n$ oe any p Binomial expression with ${}^3C_n$ P(0, 1) or 1 – P(2, 3) p = 0.15 or 0.85
	= 0.939	**A1** [5]	

259. 9709_s16_qp_61 Q: 4

A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find E(X) and Var(X). [6]

Answer:

	[P(X = 0)] = P(B, B) = 5/7×4/6 = 10/21	**M1**	Attempt to find P(0) or P(1) or P(2) can be seen as P(BB) etc. or table unsimplified
	[P(X = 1)] = P(G,B) + P(B,G) = 2/7×5/6 ×2 = 10/21	**A1**	P(1) or P(BG)+P(GB) correct
	[P(X = 2] = P(G, G) = 2/7×1/6 = 1/21	**A1**	P(0) or P(2) correct must see X value
	E(X) = 0 + 10/21 + 2/21 = 4/7 (0.571)	**B1**√	Correct answer ft their probs P(1) and P(2)
	Var(X) = 0 + 10/21 + 4/21 – $(4/7)^2$ = 50/147 (0.340)	**M1** **A1** [6]	Attempt at $\Sigma x^2 p - [E(X)]^2$

260. 9709_s16_qp_62 Q: 3

A particular type of bird lays 1, 2, 3 or 4 eggs in a nest each year. The probability of x eggs is equal to kx, where k is a constant.

(i) Draw up a probability distribution table, in terms of k, for the number of eggs laid in a year and find the value of k. [3]

(ii) Find the mean and variance of the number of eggs laid in a year by this type of bird. [3]

Answer:

(i)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 \\ \hline P(x) & k & 2k & 3k & 4k \\ \hline\end{array}$	**B1**	Probability Distribution Table, either k or correct numerical values
	10k = 1	**M1**	Summing probs involving k to = 1, 3 or 4 terms
	k = 1/10	**A1** [3]	
(ii)	E(X) = 1/10 + 4/10 + 9/10 + 16/10 = 3 Var(X) = 1/10 + 8/10 + 27/10 + 64/10 – 3^2 = 1	**B1** **M1** **A1** [3]	Correct mean Correct method seen for var, their k and μ

261. 9709_s16_qp_62 Q: 4

When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than \$50 and 13% spend at least \$50.

(i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]

(ii) n people visiting the shop are chosen at random. The probability that none of them spends at least \$50 is less than 0.04. Find the smallest possible value of n. [3]

Answer:

(i)	$p = 0.66$ $X \sim \mathrm{B}(15, 0.66)$	**M1**	Bin term $^{15}C_x p^x(1-p)^{15-x}$ seen any p
	P(at least 14) = P(14, 15) = $^{15}C_{14}\,(0.66)^{14}(0.34) + (0.66)^{15}$	**M1**	Unsimplified correct expression for P(14,15)
	= 0.0171	**A1** [3]	
(ii)	$(0.87)^n < 0.04$	**M1**	Eqn involving 0.87, power of n, 0.04 only
		M1	Solving by logs or trial and error(can be implied). Must be exponential equation
	$n = 24$	**A1** [3]	

262. 9709_s16_qp_63 Q: 3

Two ordinary fair dice are thrown. The resulting score is found as follows.

- If the two dice show different numbers, the score is the smaller of the two numbers.
- If the two dice show equal numbers, the score is 0.

(i) Draw up the probability distribution table for the score. [4]

(ii) Calculate the expected score. [2]

Answer:

(i)	P(0) = 6/36, P(1) = 10/36, P(2) = 8/36	**B1**	Table oe seen with 0, 1, 2, 3, 4, 5 (6 if P(6) = 0)
		B1	Any three probs correct
		M1	$\Sigma\, p = 1$ and at least 3 outcomes
	P(3) = 6/36, P(4) = 4/36, P(5) = 2/36	**A1** [4]	All probs correct
(ii)	mean score = (0×6+1×10 +16 +18 +16+10)/36	**M1**	Using Σxp (unsimplified) on its own – condone $\Sigma\, p$ not =1
	= 70/36 (35/18, 1.94)	**A1** [2]	

263. 9709_w16_qp_61 Q: 1

The random variable X is such that $X \sim \mathrm{N}(20, 49)$. Given that $\mathrm{P}(X > k) = 0.25$, find the value of k. [3]

Answer:

	$z = 0.674$	M1		±0.674 seen
	$0.674 = \dfrac{k-20}{7}$	M1		Standardising no cc, no sq, no sq rt
	$k = 24.7$	A1	[3]	

264. 9709_w16_qp_61 Q: 2

Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

Answer:

<table>
<tr><td></td><td><table><tr><td>diff</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>prob</td><td>6/36</td><td>10/36</td><td>8/36</td><td>6/36</td><td>4/36</td><td>2/36</td></tr></table></td><td>B1</td><td></td><td>0, 1, 2, 3, 4, 5 seen in table heading or considering all different differences</td></tr>
<tr><td></td><td></td><td>M1</td><td></td><td>Attempt at finding prob of any difference</td></tr>
<tr><td></td><td></td><td>A1</td><td></td><td>1 correct prob</td></tr>
<tr><td></td><td>Expectation = (0+10+16+18+16+10)/36
= 70/36
= 1.94</td><td>M1

A1</td><td>

[5]</td><td>Probs summing to 1</td></tr>
</table>

265. 9709_w16_qp_61 Q: 3

Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

(ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]

(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

Answer:

(i)	$0.9 \times 0.95 \times 0.85 \times 0.1 = 0.0727$	B1	[1]	
(ii)	P(0, 1, 2)	M1		Bin term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$ $p < 1$, $x \neq 0$
	$= (0.9)^{12} + {}^{12}C_1\,(0.1)(0.9)^{11} + {}^{12}C_2\,(0.1)^2(0.9)^{10}$	M1		Bin expression $p = 0.1$ or 0.9, $n = 12$, 2 or 3 terms
	$= 0.889$	A1	[3]	
(iii)	$X \sim B(50, 0.85)$	M1		50 × 0.85 seen oe can be implied
	Expectation = 50 × 0.85 (= 42.5) Var = 50 × 0.85 × 0.15 (= 6.375)	A1	[2]	Correct unsimplified mean and var

266. 9709_w16_qp_62 Q: 2

Noor has 3 T-shirts, 4 blouses and 5 jumpers. She chooses 3 items at random. The random variable X is the number of T-shirts chosen.

(i) Show that the probability that Noor chooses exactly one T-shirt is $\frac{27}{55}$. [3]

(ii) Draw up the probability distribution table for X. [4]

Answer:

<table>
<tr><td>(i)</td><td>P(1 T-shirt) = $\frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$
= 27/55 AG

OR 3/12×9/11×8/10×^{3}C$_1$oe

= 27/55 AG</td><td>B1
B1
B1

M1
M1
A1</td><td>

[3]</td><td>Correct num unsimplified
Correct denom unsimplified
Answer given, so process needs to be convincing

Mult 3 probs diff denoms (not a/3 x b/4 x c/5)
Mult by ^{3}C$_1$ oe
Answer given, so process needs to be convincing</td></tr>
<tr><td>(ii)</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>84/220</td><td>27/55</td><td>27/220</td><td>1/220</td></tr></table></td><td>B1

B1
B1
B1√</td><td>

[4]</td><td>0, 1, 2, 3 only seen in top line (condone additional values if Prob stated as 0)

One correct prob, correctly placed in table
One other correct prob, correctly placed in table
One other correct prob ft $\Sigma p = 1$, 4 values in table</td></tr>
</table>

267. 9709_w16_qp_63 Q: 2

A fair triangular spinner has three sides numbered 1, 2, 3. When the spinner is spun, the score is the number of the side on which it lands. The spinner is spun four times.

(i) Find the probability that at least two of the scores are 3. [3]

(ii) Find the probability that the sum of the four scores is 5. [3]

Answer:

(i)	$p = 1/3$ P(⩾2) = 1 – P(0, 1) = 1 – $(2/3)^4$ – ^{4}C$_1(1/3)(2/3)^3$ or P(2,3,4) = ^{4}C$_2(1/3)^2(2/3)^2$ + ^{4}C$_3(1/3)^3(2/3)$+$(1/3)^4$ = $\frac{11}{27}$, 0.407	 **M1** **M1** **A1**	 [3]	 Bin term $^4C_x p^x(1-p)^{4-x}$ $0 < p < 1$ Correct unsimplified answer
(ii)	P(sum is 5) = P(1, 1, 1, 2) ×4 = $(1/3)^4 \times 4$ = $\frac{4}{81}$, 0.0494	**M1** **M1** **A1**	 [3]	1, 1, 1, 2 seen or 4 options Mult by $(1/3)^4$

268. 9709_s15_qp_62 Q: 1

A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

Answer:

	P(3, 4, 5) =	M1	Bin expression of form $^{10}C_x\,(p)^x(1-p)^{10-x}$ any x any p
	$^{10}C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7 + {}^{10}C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6 + {}^{10}C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^5$	A1	Correct unsimplified answer accept (0.17, 0.83), (0.16, 0.84), (0.16, 0.83), (0.17, 0.84) or more accurate
		A1 **3**	Correct answer
	= 0.222		

269. 9709_s15_qp_62 Q: 5

A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

Answer:

<table>
<tr><td>(i)</td><td>P(2Es 1O) = $\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times{}^3C_2 = \frac{3}{5}$ (0.6)</td><td>M1
M1
A1 3</td><td>5×4×3 seen in denom
Mult a prob by $^3C_2$ oe
Correct answer</td></tr>
<tr><td></td><td>OR
P(2Es 1O) $= \frac{^3C_2\times{}^2C_1}{^5C_3} = \frac{6}{10}$</td><td>M1</td><td>$^3C_x$ or $^yC_2$ or $^2C_1$ oe seen mult by $k \geqslant 1$ in num</td></tr>
<tr><td></td><td>= 0.6</td><td>M1
A1</td><td>$^5C_3$ seen in denom
Correct answer</td></tr>
<tr><td></td><td>OR
241, 247, 261, 267, 461, 467 = 6 options
124 126 127 146 147 167 246 247 267 467</td><td>M1
M1</td><td>List at least 3 of 241, 247, 261, 267, 461, 467
$^5C_3$ or list to get all 10 options in denom see below</td></tr>
<tr><td></td><td>Prob = 6/10</td><td>A1</td><td>Correct answer</td></tr>
<tr><td>(ii)</td><td>124 126 127 146 147 167
246 247 267 467
<table><tr><td>s</td><td>1</td><td>2</td><td>4</td></tr><tr><td>P(S =s)</td><td>6/10</td><td>3/10</td><td>1/10</td></tr></table></td><td>M1
A1
B1
B1
B1 5</td><td>Attempt at listing with at least 7 correct
All correct and no others or all 60
1, 2, 4 only seen in top row
Any two correct
All correct</td></tr>
</table>

270. 9709_s15_qp_63 Q: 2

When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

Answer:

<table>
<tr><td>(i)</td><td><table><tr><td></td><td>Kitchen mess</td><td>Kitchen not mess</td><td>Total</td></tr><tr><td>On time</td><td>1/10</td><td>1/10</td><td></td></tr><tr><td>Not on time</td><td>1/2</td><td></td><td>4/5</td></tr><tr><td>Total</td><td>3/5</td><td>4/10</td><td></td></tr></table></td><td>B1
B1
B1 [3]</td><td>All values may be decimals or %
2 probabilities correct
2 further probabilities correct
2 further probabilities correct</td></tr>
<tr><td>(ii)</td><td>P(not on time given kitchen mess) = $\frac{1/2}{3/5}$
= 5/6 o.e.</td><td>M1
A1 [2]</td><td>A cond prob fraction seen (using corresponding combined outcomes and total)
FT from their values, 3sf or better, <1, 3/5ft<1</td></tr>
</table>

271. 9709_s15_qp_63 Q: 4

A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

Answer:

<table>
<tr><td>(i)</td><td>P(1 W) = 6/9×3/8 + 3/9×6/8
= ½ AG
OR $\frac{{}^6C_1 \times {}^3C_1}{{}^9C_2}$
= ½ AG</td><td>M1
A1 [2]
M1
A1</td><td>summing 2 two-factor probs (condone replacement) not ½×½ + ½×½
Correct answer, fully justified
Using combinations consistent, correct format
Correct answer, fully justified</td></tr>
<tr><td>(ii)</td><td>$P(\overline{W},\overline{W})$ = 3/9×2/8 = 6/72 (1/12)
$P(W,W)$ = 6/9×5/8 = 30/72 (5/12)<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>1/12</td><td>1/2</td><td>5/12</td></tr></table></td><td>B1
B1
B1 √ [3]</td><td>Distribution table with 0,1,2 only
$P(W,W)$ or $P(\overline{W},\overline{W})$ correct
$P(W,W) + P(\overline{W},\overline{W}) = 0.5$</td></tr>
<tr><td>(iii)</td><td>E(X) = 16/12 (4/3) (1.33) isw</td><td>B1 [1]</td><td>Condone 1(.3) if correct working seen, nfww</td></tr>
</table>

272. 9709_w15_qp_61 Q: 1

In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

Answer:

	$p = 0.76$ P(fewer than 10) = 1 – P(10, 11) $= 1 - (0.76)^{10}(0.24)\,{}^{11}C_{10} - (0.76)^{11}$ = 1 – 0.219 = 0.781	**M1** **M1** **M1** **A1** [4]	Any binomial term ${}^{11}C_x p^x(1-p)^{11-x}$, $0 < p < 1$ Any binomial term ${}^nC_x(0.76)^x(0.24)^{n-x}$ 1 – P(10, 11) oe binomial expression Correct answer

273. 9709_w15_qp_61 Q: 4

(a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

(b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]

Answer:

(i)	$\bar{x} = 80 - 147/30 = 80 - 4.9$ $= 75.1$ $sd = \sqrt{\left(\frac{952}{30} - \left(\frac{147}{30}\right)^2\right)} = \sqrt{7.72\ldots}$ sd = 2.78	**M1** **A1** **M1** **A1** [4]	For –147/30 oe seen Correct answer 952/30 – (± their coded mean)^2 Correct answer
(ii)	$P(x > 160) = P\left(z > \frac{160 - 148.6}{18.5}\right)$ $= P(z > 0.616)$ $= 1 - 0.7310$ $= 0.269$	**M1** **M1** **A1** [3]	Standardising no cc no sq rt 1 – Φ Correct answer

274. 9709_w15_qp_61 Q: 6

Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

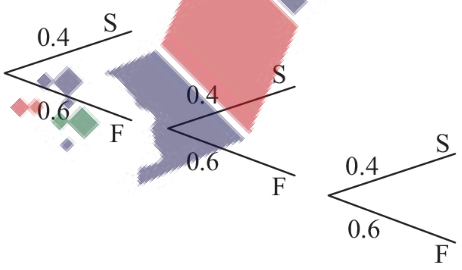
(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

Answer:

<table>
<tr><td>(i)</td><td> </td><td>M1
A1
A1 [3]</td><td>3 pairs S (bank, log in, success oe) and F oe seen no extra bits.
Exactly 3 pairs, must be labelled
Correct diagram with all probs correct</td></tr>
<tr><td>(ii)</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>0.4</td><td></td><td>0.144</td><td>0.216</td></tr></table></td><td>B1
M1

A1
B1 [4]</td><td>P(0) correct
Multiplying two of more factors of 0.4 and 0.6
One more correct prob
One more correct prob</td></tr>
<tr><td>(iii)</td><td>$E(X) = 0.24 + 2 \times 0.144 + 3 \times 0.216$
$= 1.176$ (1.18)</td><td>M1
A1 [2]</td><td>Using $\Sigma p_i x_i$
Correct answer</td></tr>
</table>

275. 9709_w15_qp_62 Q: 3

One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

Answer:

(i)	$\frac{1}{4}$	**B1** 1	
(ii)	$\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)=\frac{81}{1024}=0.0791$	**M1** **A1** 2	Expression of form $p^4(1-p)$ only, $p = 1/4$ or $3/4$ Correct answer
(iii)	P(all diff) $=\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times 4!$ $=\frac{3}{32}$ (0.0938) OR $1\times\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{3}{32}$	**M1** **M1** **A1** 3	4! on numerator seen mult by k $\geqslant$ 1 or $3\times2\times1$ on num oe, must be in a fraction. 4^4 on denom or 4^3 on denom with the $3\times 2\times 1$ Correct answer

276. 9709_w15_qp_62 Q: 6

A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	−3	−2			
	−2			1	
	−1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find Var(X). [3]

(iv) Find the probability that X is even, given that X is positive. [2]

Answer:

(i)	Spinner A $$\begin{array}{cccccc} & & 1 & 2 & 3 & 3 \\ \hline & -3 & (-2) & -1 & 0 & 0 \\ \text{Spinner } B & -2 & -1 & 0 & (1) & 1 \\ & -1 & 0 & 1 & 2 & 2 \\ & 1 & 2 & 3 & 4 & 4 \end{array}$$	**B1** 1	
(ii)	$$\begin{array}{cccccccc} x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ \hline \text{prob} & \frac{1}{16} & \frac{2}{16} & \frac{4}{16} & \frac{3}{16} & \frac{3}{16} & \frac{1}{16} & \frac{2}{16} \end{array}$$	**M1** **M1** **A1** 3	Their values in (i) as the top line, seen listed in (ii) or used in part (iii) Attempt at probs seen evaluated, need at least 4 correct from their table Correct table seen
(iii)	$E(X) = 1$ $\text{Var}(X) = ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$ $= \frac{62}{16} - 1$ $= \left(\frac{23}{8}\right)$ (2.875) OR using $\Sigma p(x - \bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$ $= \frac{46}{16} = 2.875$	**M1** **M1** **A1** 3 **M1** **M1** **A1**	Attempt at E(X) from their table if $\Sigma p = 1$ Evaluating $\Sigma x^2 p$ – [their E(X)]^2 allow $\Sigma p \neq 1$ but all p's <1 Correct answer
(iv)	P(even given +ve) $= \frac{5}{9}$ OR P(even given +ve) = $\frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$ $= \frac{5}{9}$ (0.556)	**M1** **A1** 2 **M1** **A1**	Counting their even numbers and dividing by their positive numbers Correct answer Using cond prob formula not P(E) × P(+ve) need fraction over fraction accept any of $\frac{5/16 or 6/16 or 9/16}{9/16 or 10/16 or 13/16}$ Correct answer