Cambridge Assessment International Education

Cambridge Pre-U

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2023**

2 hours

* 4 1 2 5 9 2 9 4 1 2 *

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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1 **(a)** Show that $x + 1$ is a factor of $4x^3 + 8x^2 + x - 3$. [2]

(b) Hence solve the equation $4x^3 + 8x^2 + x - 3 = 0$. [4]

2 In an arithmetic progression, the first term is 32, the last term is 14 and the sum of the terms is 299. Find the number of terms and the common difference. [4]

3 Functions f and g are defined for all real values of x by $\mathrm{f}(x) = x^2 + 2$ and $\mathrm{g}(x) = 3x + 5$.

(a) State the range of f and the range of g. [2]

(b) Find the value of $\mathrm{gf}(4)$. [2]

(c) State, with a reason, which of f and g does not have an inverse. [1]

4 Solve the equation $\cos 2\theta = 2\cos\theta$ for $0° < \theta < 360°$. [5]

5 A curve is given parametrically by $x = 3t - \ln(3t)$, $y = t^3 - \ln(t^3)$, where $t > 0$. Find the exact value of t at the point on the curve at which the gradient is 3. [5]

6 **(a)** Find $\int x\mathrm{e}^{2x}\,\mathrm{d}x$. [3]

(b) Determine, in exact form, $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \cos^2(3x)\,\mathrm{d}x$. [5]

7 **(a)** Expand $\dfrac{1}{\sqrt{1+2x}}$ as a series of ascending powers of x, up to and including the term in x^3. Express each coefficient in its simplest form. [4]

(b) Hence determine the coefficient of x^2 in the expansion of $\dfrac{(2-3x)^2}{\sqrt{1+2x}}$. [3]

8 **(a)** Express $\dfrac{2x^2+1}{x(x-1)^2}$ as the sum of three partial fractions. [5]

(b) Hence show that $\int_2^4 \dfrac{2x^2+1}{x(x-1)^2}\,\mathrm{d}x = p + \ln q$, where p and q are integers to be found. [5]

9 **(a)** Give full details of a sequence of transformations which maps the graph of $y = e^x$ onto the graph of $y = e^{2-x}$. [2]

(b) Sketch, on the same diagram, the graphs of $y = e^{2-x}$ and $y = \ln(x+1)$, and hence show that the equation $e^{2-x} = \ln(x+1)$ has only one root. [3]

(c) This root may be found using the iterative formula $x_{n+1} = 2 - \ln\big(\ln(x_n + 1)\big)$. Using a starting value of 1.5, and showing the result of each iteration, find the value of this root correct to 3 significant figures. [3]

10 The equation of a curve C is $x^2 + 2xy + 3y^2 = 18$.

(a) Show that the equation of the normal to C at the point $(5, -1)$ can be written as $x = 2y + 7$. [6]

(b) Find the exact coordinates of the point where this normal meets C again. [4]

11 The fixed line L has vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$.

For each value of the parameter t, the line $M(t)$ is given by the equation $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ t \end{pmatrix}$.

(a) Determine the value of t for which

(i) L and $M(t)$ are perpendicular; [2]

(ii) L and $M(t)$ intersect. [5]

(b) The point $P\,(30, 3, 12)$ is on L and the line $M(-1)$ has equation $\mathbf{r} = \begin{pmatrix} 7 + 3\mu \\ 1 + 2\mu \\ 2 - \mu \end{pmatrix}$.

The point Q lies on $M(-1)$.

(i) Show that $(PQ)^2 = 14\mu^2 - 126\mu + 633$. [2]

(ii) Given now that Q is the point on $M(-1)$ which is closest to P, find the coordinates of Q. [3]

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