Cambridge Assessment International Education

Cambridge Pre-U

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	$f(-1) = 4(-1)^3 + 8(-1)^2 + (-1) - 3$	**M1**	Attempt $f(-1)$
	$f(-1) = -4 + 8 - 1 - 3 = 0$ hence $(x + 1)$ is a factor	**A1**	Show $f(-1) = 0$ and conclude
1(b)	$f(x) = (x + 1)(4x^2 + \dots)$	**M1**	Attempt complete division by $(x + 1)$
	$= (x + 1)(4x^2 + 4x - 3)$	**A1**	Obtain correct quadratic factor
	$(x + 1)(2x + 3)(2x - 1) = 0$	**M1**	Attempt to factorise / solve quadratic
	$x = -1, x = -\frac{3}{2}, x = \frac{1}{2}$	**A1**	Obtain three correct roots

Question	Answer	Marks	Guidance
2	$\frac{1}{2}n(32+14)=299$	**M1**	Attempt use of $\frac{1}{2}n(a+l)=299$
	$n = 13$	**A1**	Obtain $n = 13$
	$32 + 12d = 14$	**M1**	Attempt use of $a + (n - 1)d = 14$
	$d=-\frac{3}{2}$	**A1**	Obtain $d=-\frac{3}{2}$
	Alt. $32 + (n - 1)d = 14$ and $\frac{1}{2}n(64 + (n - 1)d) = 299$	**(M1)**	Both attempted
	e.g. $(n - 1)d = -18 \Rightarrow n(64 - 18) = 598$	**(M1)**	Complete solving attempt
	$n = 13$ and $d = -\frac{3}{2}$	**(A1 A1)**	
3(a)	$f(x) \geqslant 2$ or equivalent in words/set notation	**B1**	Allow y for $f(x)$, but not x
	$g(x) \in \mathbb{R}$ or equivalent in words/set notation	**B1**	Allow y for $g(x)$, but not x

Question	Answer	Marks	Guidance
3(b)	gf(4)= g(18)	**M1**	Attempt both or find gf(x) = $3x^2 + 11$ with $x = 4$
	= 59	**A1**	Obtain 59
3(c)	f has no inverse, as it is a many-to-one function **or** f is not a 1–1 function (**or** showing it so: via e.g. f(–3) = f(3) = 11)	**B1**	Accept 'f fails the *horizontal line test*' or equivalent

Question	Answer	Marks	Guidance
4	$2\cos^2\theta - 1 = 2\cos\theta$	**M1**	Attempt $\cos 2\theta = \pm 2\cos^2\theta \pm 1$
	$2\cos^2\theta - 2\cos\theta - 1 = 0$	**A1**	Obtain correct quadratic
	$\theta = \cos^{-1}\dfrac{2 \pm \sqrt{12}}{4}$ or 1.366 … / –0.366 …	**M1**	Attempt to find θ; i.e. solve quadratic and use $\cos^{-1}$ Condone no sight of positive root
	θ = 111.5°	**A1**	**cao** 1st answer
	θ = 248.5°	**A1FT**	FT 2nd answer = 360° – their 1st answer

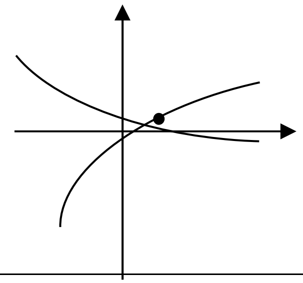
Question	Answer	Marks	Guidance
5	$\frac{dx}{dt} = 3 - \frac{1}{t}$	**B1**	
	$\frac{dy}{dt} = 3t^2 - \frac{3}{t}$	**B1**	
	$\frac{dy}{dx} = \frac{3t^2 - \frac{3}{t}}{3 - \frac{1}{t}}$	**M1**	Attempt to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
	$\frac{3t^2 - \frac{3}{t}}{3 - \frac{1}{t}} = 3 \Rightarrow 3t^2 - \frac{3}{t} = 9 - \frac{3}{t} \Rightarrow 3t^2 = 9$	**M1**	Equate to 3 and attempt to solve for t
	$t = \sqrt{3}$	**A1**	Obtain $t = \sqrt{3}$ only

Question	Answer	Marks	Guidance
6(a)	$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$	**M1**	Attempt integration by parts
		A1	Obtain correct expression (allow unsimplified)
	$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$	**A1**	Obtain correct final integral (condone missing + c)
6(b)	$\int \cos^2 3x\ dx = \frac{1}{2}\int (\cos 6x + 1)\ dx$	**B1**	Correct double angle formula seen or implied
	$= \frac{1}{2}\left(\frac{1}{6}\sin 6x + x\right)$	**M1**	Attempt integration of $a\cos 6x$ and b
	$= \frac{1}{12}\sin 6x + \frac{1}{2}x$	**A1**	**FT** Obtain correct integrals: $\frac{1}{6}a\sin 6x$ and bx
	$I = \left(\frac{1}{12}\sin\pi + \frac{1}{12}\pi\right) - \left(\frac{1}{12}\sin\frac{1}{2}\pi + \frac{1}{24}\pi\right)$	**M1**	Attempt use of limits (correct order and subtracted)
	$= \frac{1}{24}\pi - \frac{1}{12}$	**A1**	Any exact form

Question	Answer	Marks	Guidance
7(a)	$(1+2x)^{-\frac{1}{2}} = 1 + (-\tfrac{1}{2})(2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(2x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6}(2x)^3$	**B1**	Obtain $1 - x + \ldots$
		M1	Full attempt at $\geqslant 1$ of quadratic/cubic terms
	$= 1 - x + \tfrac{3}{2}x^2 - \tfrac{5}{2}x^3$	**A1**	Obtain correct coefficient of x^2
		A1	Obtain correct coefficient of x^3
7(b)	$\left(1 - x + \tfrac{3}{2}x^2 - \tfrac{5}{2}x^3\right)\left(4 - 12x + 9x^2\right)$	**M1**	Multiply **(a)**'s answer by correct full quadratic
	The x^2 term is $(1 \times 9x^2) + (-x \times -12x) + (\tfrac{3}{2}x^2 \times 4)$	**M1**	Identifying the 3 ways to get an x^2 term
	Coefficient is 27	**A1**	Correct coefficient (allow $27x^2$)

Question	Answer	Marks	Guidance
8(a)	$\frac{2x^2+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$	**M1**	Correct structure for partial fractions
	$A(x-1)^2 + Bx(x-1) + Cx = 2x^2 + 1$	**M1**	Set up and attempt to use identity (or "Cover-Up")
	$A = 1 \quad B = 1 \quad C = 3$	**A1 A1 A1**	Any one correct constant $\Rightarrow$ **M** earned
			Note that the (otherwise correct) form $\frac{A}{x} + \frac{Bx+C}{(x-1)^2}$ scores **M0 M1** (for the attempt to evaluate any of the constants) **A1** (for A = 1) **A0 A0** (B = 1, C = 2)

Question	Answer	Marks	Guidance
8(b)	$\int f(x)dx = \ln\lvert x\rvert + \ln\lvert x-1\rvert - 3(x-1)^{-1}$	**M1**	Attempt at integration ($\geqslant$ 1 term correct)
		A1FT	**Three** correct integral terms (FT non-zero A, B, C) Modulus brackets not required
	(ln4 + ln3 – 1) – (ln2 + ln1 – 3)	**M1**	Attempt use of limits (correct order and subtracted)
	$2 + \ln\frac{4\times3}{2}$	**M1**	Some simplification using log law(s)
	2 + ln6	**A1**	
	Alt. FT from $\frac{1}{x}+\frac{x+2}{(x-1)^2}$ they will need to use (e.g.) a substn. $\int\frac{x+2}{(x-1)^2}dx = (u = x-1)\ \int\frac{u+3}{u^2}du = \int\left(\frac{1}{u}+3u^{-2}\right)du$ **M1** Then $\int f(x)dx = \ln\lvert x\rvert + \ln\lvert x-1\rvert - 3(x-1)^{-1}$ **A1** **M1** for correct use of limits (x only or x, u separately) and final **M1 A1** as above		Allow for $\ln\lvert u\rvert - 3u^{-1}$ for 2nd- and 3rd-terms

Question	Answer	Marks	Guidance
9(a)	* Translate $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ then Reflect in y-axis **OR** Translate $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ then Reflect in $x = 2$ * Reflect in y-axis then Translate $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ * Reflect in y-axis and Stretch by e^2 || to the y-axis (either order) * Reflect in $x = 1$	**M1**	Translation and reflection for the **M** **OR** Reflection and translation for the **M** **OR** Reflection and stretch for the **M** This case is **B0** or **B2**
		A1	Correct details for their order of transformations
9(b)		**B1**	Correct graph of $y = e^{2-x}$, with $(0, e^2)$ identified
		B1	Correct graph of $y = \ln(x + 1)$, passing through $(0, 0)$ with asymptote of $x = -1$ soi
		B1	Identify one point of intersection hence one root – can follow B0 as long as both graphs of correct shape
9(c)	2.087…	**B1**	Correct first iterate, from starting with 1.5
	… 1.880, 1.944, 1.923, 1.930, 1.928, 1.928…	**M1**	Correct iteration process at least 3 times Allow working to 3sf
	Hence root is 1.93	**A1**	Obtain 1.93 (must be 3sf)

Question	Answer	Marks	Guidance
10(a)	$2x+\left(2y+2x\frac{dy}{dx}\right)+6y\frac{dy}{dx}=0$	**M1**	Attempt implicit differentiation including use of the product rule
		A1	Obtain correct derivative, unsimplified
	$\frac{dy}{dx}=-\frac{2x+2y}{2x+6y}=-\frac{4}{2}=-2$ or $10+\left(-2+10\frac{dy}{dx}\right)-6\frac{dy}{dx}=0$ and rearrangement for $\frac{dy}{dx}$	**M1**	Substitute $(5, -1)$ and attempt to find $\frac{dy}{dx}$, either algebraically or numerically
	$m'=-\frac{1}{-2}=\frac{1}{2}$	**M1**	Use $mm'=-1$ to find gradient of normal
	$y+1=\frac{1}{2}(x-5)$	**M1**	Attempt equation of normal
	$x = 2y + 7$	**A1**	**A.G.** Legitimately obtain given equation of normal
10(b)	$(2y + 7)^2 + 2y(2y + 7) + 3y^2 = 18$	**M1**	Eliminate one variable (hopefully x)
	$11y^2 + 42y + 31 = 0$	**A1**	Obtain correct three term quadratic **or** $11x^2 - 70x + 75 = 0$
	$(11y + 31)(y + 1) = 0$	**M1**	Attempt to solve quadratic **or** $(11x - 15)(x - 5) = 0$
	$y=-\frac{31}{11}$ so point of intersection is $\left(\frac{15}{11},-\frac{31}{11}\right)$	**A1**	Obtain both correct coordinates in exact form

Question	Answer	Marks	Guidance
11(a)(i)	$\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 2 \\ t \end{pmatrix} = 0$	**M1**	Scalar product of direction vectors set to zero
	$\Rightarrow 12 + 2 + t = 0 \Rightarrow t = -14$	**A1**	
11(a)(ii)	$4\lambda + 2 = 3\mu + 7$ $\lambda - 4 = 2\mu + 1$	**B1**	Correctly stated system of equations to be solved
	Eliminating one variable	**M1**	Method for solving simultaneously for λ, μ
	$\lambda = -1$ and $\mu = -3$	**A1**	
	Substituting their λ, μ into 3rd eqn. (the **k**-component)	**M1**	
	$t = -\frac{2}{3}$	**A1**	
11(b)(i)	The components of $\begin{pmatrix} 3\mu - 23 \\ 2\mu - 2 \\ -\mu - 10 \end{pmatrix}$ squared and added	**M1**	i.e. $\overrightarrow{PQ}$ attempted *and* the square of its magnitude
	Expanding $(3\mu - 23)^2 + (2\mu - 2)^2 + (\mu + 10)^2$ $= (9\mu^2 - 138\mu + 529) + (4\mu^2 - 8\mu + 4) + (\mu^2 + 20\mu + 100)$ $= 14\mu^2 - 126\mu + 633$	**A1**	**A.G.** legitimately obtained

Question	Answer	Marks	Guidance
11(b)(ii)	Using calculus: $d(PQ^2)/d\mu = 28\mu - 126 = 0$ **Or** by completing the square: $14(\mu - 4.5)^2 + 349.5$	**M1**	Attempt to find the value of μ which minimises PQ^2
	$\mu = 4.5$	**A1**	In the completing the square method, condone incorrect + c (since irrelevant to the Qn.)
	$Q = (20.5, 10, –2.5)$	**A1**	