Algebra

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Algebra
Booklet	Question Paper

Time Allowed: 106 minutes

Score: /88

Percentage: /100

Grade Boundaries:

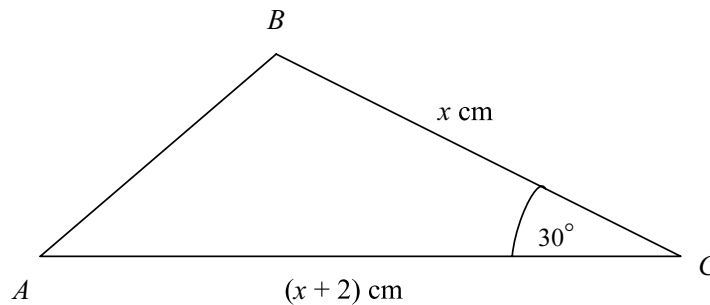**1**   **(i)** Show that $x = 2$ is a root of the equation $2x^3 - x^2 - 15x + 18 = 0$.   [1

     (ii) Hence solve the equation $2x^3 - x^2 - 15x + 18 = 0$.   [5

2   **(i)** Express $\dfrac{8x - 1}{(2x - 1)(x + 1)}$ in the form $\dfrac{A}{2x - 1} + \dfrac{B}{x + 1}$ where A and B are constants.   [4]

    (ii) Hence show that $\displaystyle\int_2^5 \dfrac{8x - 1}{(2x - 1)(x + 1)}\,dx = \ln 24$.   [5]

3

The diagram shows a triangle ABC in which angle $C = 30°$, $BC = x$ cm and $AC = (x + 2)$ cm. Given that the area of triangle ABC is 12 cm^2, calculate the value of x.   [5]

4 Solve the inequality $|2x - 1| < 3$.   [3]

5   Let $f(x) = x^2 + kx + 4$, where k is a constant.

    (i) Find an expression for the discriminant of f in terms of k. [2]

    (ii) Hence find the range of values of k for which the equation $f(x) = 0$ has two distinct real roots. [3]

6   Let $f(x) = x^4 - 4x^3 - 10x^2 + 28x - 15$.

    (i) Show that $x = 1$ is a root of the equation $f(x) = 0$. [2]

    (ii) Find the quotient and remainder when $f(x)$ is divided by $x - 5$. [4]

    (iii) Factorise $f(x)$ fully and hence sketch the graph of $y = f(x)$. [5]

7   **(i)** Prove the identity $\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2} \equiv \dfrac{-2hx - h^2}{x^2(x+h)^2}$. [3]

    (ii) Given that $f(x) = x^{-2}$, use differentiation from first principles to find an expression for $f'(x)$. [3]

8

By first factorising completely $x^3 + x^2 - 5x + 3$, find $\displaystyle\int \frac{2x^2 + x + 1}{x^3 + x^2 - 5x + 3} \, dx$. [12]

9

(i) Express $x^2 + 2x - 3$ in the form $(x + a)^2 + b$, where a and b are integers to be found. [2]

(ii) Sketch the graph of $y = x^2 + 2x - 3$ giving the coordinates of the vertex and of any intersections with the coordinate axes. [5]

10 Let $f(x) = x^3 - 3x^2 - 13x + 15$.

(i) Show that $f(1) = 0$ and hence factorise $x^3 - 3x^2 - 13x + 15$ completely. [4]

(ii) Hence solve the equation $x^3 - 3x^2 - 13x + 15 = 0$. [2]

11 Solve the equation $3 + 2x = |7 - 4x|$. [3]

12 (i) Show that $x = 4$ is a root of $x^3 - 12x - 16 = 0$. [2]

(ii) Hence completely factorise the expression $x^3 - 12x - 16$. [3]

13   **(i)** Express $\dfrac{x-1}{x^2+2x+1}$ in the form $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$, where A and B are integers.   [2]

  (ii) Find the quotient and remainder when $2y^2+1$ is divided by $y+1$.   [2]

  (iii) A curve in the x-y plane passes through the point $(0, 2)$ and satisfies the differential equation

$$(2y^2+1)(x^2+2x+1)\frac{\mathrm{d}y}{\mathrm{d}x}=(x-1)(y+1).$$

  By solving the differential equation find the equation of the curve in implicit form.   [6]