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Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5 [Probability & Statistics 1]

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Format Type B:
Each question is followed by its answer scheme

Chapter 5

The normal distribution

277. 9709_m22_qp_52 Q: 4

The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

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The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. [3]

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The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$P(46 < X < 62) = P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$	**M1**	46 or 62, 55 and 6 substituted into ±standardisation formula once. Condone 6^2 and continuity correction ±0.5
	$= P\left(-1.5 < Z < \frac{7}{6}\right)$	**B1**	Both standardisation values correct, accept unsimplified
	$\left[=\Phi\left(\frac{7}{6}\right)-(1-\Phi(1.5))\right]$ $= 0.8784 + (0.9332 - 1)$	**M1**	Calculating the appropriate area from stated Φs of *z*-values, must be probabilities.
	0.812	**A1**	$0{\cdot}8115 < p \leqslant 0{\cdot}812$
		4	
(b)	$z = \pm 0.674$	**B1**	CAO, critical *z*-value
	$\frac{36-42}{\sigma} = -0.674$	**M1**	36 and 42 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a *z*-value
	$\sigma = 8.9[0]$	**A1**	WWW. Only dependent on M.
		3	

Question	Answer	Marks	Guidance
(c)	P(male < 46) = 1−*their* 0.9332 = 0.0668	**M1**	FT value from part (a) or Correct: $1-\Phi\left(\frac{46-55}{6}\right)$,condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found. Condone unsupported correct value stated.
	P(female < 46) = $P(Z < \frac{46-42}{their\,8.90})[=\Phi(0.449)]$ $=0.6732$	**M1**	46, 42 and *their* 4(b) σ (or correct σ) substituted in ±standardisation formula, condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found. Condone $\frac{4}{their\,8.90}$.
	P(both) = 0.0668 ×0.6732	**M1**	Product of *their* 2 probabilities (0 < both < 1) Not 0.25 or *their* final answer to **4(a)** used.
	0.0450 or 0.0449	**A1**	$0{\cdot}0449 \leqslant p \leqslant 0{\cdot}0450$
		4	

278. 9709_s22_qp_51 Q: 5

The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

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The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

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(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

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Answer:

Question	Answer	Marks	Guidance
(a)	$P(X<6) = P(Z < \frac{6-5.2}{1.5}) = P(Z < 0.5333)$	M1	6, 5.2, 1.5 substituted into ± standardisation formula, condone 1.5^2, continuity correction ±0.5
	0.703	A1	
		2	
(b)	$z_1 = \frac{3-\mu}{\sigma} = -1.329$	B1	$1.328 < z_1 \leqslant 1.329$ or $-1.329 \leqslant z_1 < -1.328$
	$z_2 = \frac{8-\mu}{\sigma} = 0.878$	B1	$0.877 < z_2 \leqslant 0.878$ or $-0.878 \leqslant z_2 < -0.877$
	Solve to find at least one unknown: $\frac{3-\mu}{\sigma} = -1.329$ $\frac{8-\mu}{\sigma} = 0.878$	M1	Use of the ± standardisation formula once with μ, σ, a z-value (not 0.8179, 0.7910, 0.5367, 0.5753, 0.19, 0.092 etc.) and 3 or 8, condone continuity correction but not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve their two equations in μ and σ
	$\sigma = 2.27, \quad \mu = 6.01$	A1	$2.26 \leqslant \sigma \leqslant 2.27$, $6.01 \leqslant \mu \leqslant 6.02$
		5	

Question	Answer	Marks	Guidance
(c)	$[P(Z<-1) + P(Z>1)]\ \Phi(1) - \Phi(-1) =$	M1	Identify 1 and –1 as the appropriate z-values.
	$= 2 - 2\ \Phi(1)$ $= 2 - 2 \times 0.8413$	M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.3174	A1	Accept AWRT 0.317
	Number of leaves: $2000 \times 0.3174 = 634.8$ so 634 or 635	B1 FT	FT *their* 4 s.f. (or better) probability, final answer must be positive integer no approximation or rounding stated
		4	

279. 9709_s22_qp_52 Q: 4

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$[P(1.98 < X < 2.03) =]P(\frac{1.98-2.02}{0.03} < z < \frac{2.03-2.02}{0.03})$ $[= P(-1.333 < z < 0.333)]$	M1	Use of ±standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ±0.005, not √0.03.
	$[= \Phi(0.333) - (1 - \Phi(1.333))]$ $= 0.6304 + 0.9087 - 1$	M1	Calculating the appropriate probability area from *their* z-values. (or $0.6304 - 0.09121$ or $(0.9087 - 0.5) + (0.6304 - 0.5)$ etc)
	0.539	A1	$0.539 \leqslant z < 0.5395$ Only dependent upon 2nd M mark. If M0 scored **SC B1** for $0.539 \leqslant z < 0.5395$.
		3	
(b)	$[P(X > 2.6) = \frac{134}{5000} = 0.0268]$ $[P(X<2.6) = 1 - 0.0268 =]\ 0.9732$	B1	0.9732 or $\frac{4866}{5000}$ or $\frac{2433}{2500}$ seen.
	$\frac{2.6-2.55}{\sigma} = 1.93$	M1	Use of ±standardisation formula with 2.6 and 2.55 substituted, no $\sigma^2, \sqrt{\sigma}$ or continuity correction.
		M1	*Their* standardisation formula with values substituted equated to z-value which rounds to ±1.93.
	$\sigma = 0.0259$	A1	AWRT 0.0259 or $\frac{5}{193}$. If M0 earned, **SC B1** for correct final answer.
		4	

280. 9709_s22_qp_52 Q: 5

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	[P(10, 11, 12) =] ${}^{12}C_{10}\,0.72^{10}\,0.28^{2} + {}^{12}C_{11}\,0.72^{11}\,0.28^{1} + {}^{12}C_{12}\,0.72^{12}\,0.28^{0}$	M1	One term ${}^{12}C_x\,p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	$= 0.193725 + 0.0905726 + 0.0194084$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	[1 – P(0,1,2,3,4,5,6,7,8,9) =] $1 - ({}^{12}C_0\,0.72^{0}\,0.28^{12} + {}^{12}C_1\,0.72^{1}\,0.28^{11} + {}^{12}C_2\,0.72^{2}\,0.28^{10} + {}^{12}C_3\,0.72^{3}\,0.28^{9} + {}^{12}C_4\,0.72^{4}\,0.28^{8} + {}^{12}C_5\,0.72^{5}\,0.28^{7} + {}^{12}C_6\,0.72^{6}\,0.28^{6} + {}^{12}C_7\,0.72^{7}\,0.28^{5} + {}^{12}C_8\,0.72^{8}\,0.28^{4} + {}^{12}C_9\,0.72^{9}\,0.28^{3})$	M1	One term ${}^{12}C_x\,p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
		3	
(b)	Mean = $[0.52 \times 90 =]\,46.8$, var $= [0.52 \times 0.48 \times 90] = 22.464$	B1	46.8 and 22.464 or 22.46 seen, allow unsimplified, ($4.739 < \sigma \leqslant 4.740$ imply correct variance).
	$[P(X < 40) =]\ P\left(z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$	M1	Substituting *their* mean and *their* variance into ±standardisation formula (any number for 39.5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	$= [P(Z < -1.540)] = 1 - 0.9382$	M1	Appropriate area Φ, from final process, must be probability.
	0.0618	A1	$0.06175 < p \leqslant 0.0618$
		5	

281. 9709_s22_qp_53 Q: 5

Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

Farmer Jones sells the apples to the supermarket at $0.24 each. He sells apples that weigh more than 205 grams to a local shop at $0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

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Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$[P(142 < X < 205)] = P\left(\frac{142-170}{25} < z < \frac{205-170}{25}\right)$	M1	Use of ± standardisation formula once substituting 170, 25 and either 142 or 205 appropriately.. Condone 25^2 and continuity correction ±0.5.
	$P(-1.12 < z < 1.4)$	A1	Both correct. Accept unsimplified.
	$\Phi(1.4) - (1 - \Phi(1.12)) = 0.9192 + 0.8686 - 1$	M1	Calculating the appropriate area from stated phis of z-values.
	0.788	A1	AWRT, not from wrong working
		4	
(b)	$P(X > 205) = 1 - 0.9192 = 0.0808$	B1 FT	Correct or FT from part **5(a).**
	$(0.0808 \times 0.30 + \textit{their}\ 0.788 \times 0.24) \times 20000$	M1	Correct or *their* $0.0808 \times 0.30 \times k$ + *their* $0.788 \times 0.24 \times k$, k positive integer.
	[$]4266.24	A1	$4265 < \text{income} \leqslant 4270$, not from wrong working
		3	
(c)	$[P(Z > \frac{w-182}{20}) = 0.72]$	B1	$0.5828 \leqslant z \leqslant 0.583$ or $-0.583 \leqslant z \leqslant -0.5828$ seen.
	$\frac{w-182}{20} = -0.583$	M1	182 and 20 substituted in ± standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 170$	A1	$170 \leqslant w < 170.35$
		3	

282. 9709_m21_qp_52 Q: 3

The time spent by shoppers in a large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. [3]

88% of shoppers spend more than t minutes in the shopping centre.

(b) Find the value of t. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$P\left(\left(\frac{85-96}{18}\right) < z < \left(\frac{100-96}{18}\right)\right)$	M1	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$P(-0.6111 < z < 0.2222)$ $= \Phi(0.2222) + \Phi(0.6111) - 1$ $= 0.5879 + 0.7294 - 1$	M1	Appropriate area Φ, from final process, must be probability. Use of $(1 - z)$ implies M0.
	0·317	A1	Final answer which rounds to 0·317.
		3	

Question	Answer	Marks	Guidance
(b)	$z = \pm 1{\cdot}175$	B1	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175 = \frac{t-96}{18}$	M1	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	A1	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

283. 9709_m21_qp_52 Q: 7

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

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(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

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On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

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(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

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Answer:

Question	Answer	Marks	Guidance
(a)(i)	$\left[\frac{104+31}{400}=\right]\frac{135}{400}, \frac{27}{80}, 0{\cdot}3375$	B1	Evaluated, exact value.
		1	
(a)(ii)	**Method 1**		
	$P(M)=\frac{180}{400}, 0{\cdot}45\ P(S)=\frac{135}{400}, 0{\cdot}3375\ P(M\cap S)=\frac{31}{400}, 0{\cdot}0775$ $\frac{180}{400}\times\frac{135}{400}=\frac{243}{1600}, 0.151875\neq\frac{31}{400}$ so NOT independent	M1	*Their* P(*M*) × *their* P(*S*) seen, accept unsimplified.
		A1	P(*M*), P(*S*) and P(*M* ∩ *S*) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$P(M\cap S)=\frac{31}{400}\quad P(S)=\frac{135}{400}\quad P(M)=\frac{180}{400}$ $P(M\mid S)=\frac{\frac{31}{400}}{\frac{135}{400}}=\frac{31}{135}, 0.2296\ldots\neq\frac{180}{400}$ so NOT independent	M1	$[P(M\mid S)=]\frac{their\ P(M\cap S)}{their\ P(S)}$ (oe) seen, accept unsimplified.
		A1	P(*M*), P(*S*) and P(*M* ∩ *S*) notation seen, numerical comparison and correct conclusion, WWW.
		2	

Question	Answer	Marks	Guidance
(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\, 0{\cdot}3^0\, 0{\cdot}7^{10} + {}^{10}C_1\, 0{\cdot}3^1\, 0{\cdot}7^9$ $+ {}^{10}C_2\, 0{\cdot}3^2\, 0{\cdot}7^8)$	M1	${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
	= 1 – (0·028248 + 0·121061 + 0·233474)	A1	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	= 0·617	A1	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\, 0{\cdot}3^3\, 0{\cdot}7^7 + {}^{10}C_4\, 0{\cdot}3^4\, 0{\cdot}7^6 + {}^{10}C_5\, 0{\cdot}3^5\, 0{\cdot}7^5$ $+ {}^{10}C_6\, 0{\cdot}3^6\, 0{\cdot}7^4 + {}^{10}C_7\, 0{\cdot}3^7\, 0{\cdot}7^3 + {}^{10}C_8\, 0{\cdot}3^8\, 0{\cdot}7^2$ $+ {}^{10}C_9\, 0{\cdot}3^9\, 0{\cdot}7^1 + {}^{10}C_{10}\, 0{\cdot}3^{10}\, 0{\cdot}7^0$	M1	${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	= 0·617	A1	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	

Question	Answer	Marks	Guidance
(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	B1	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$P(X < 32) = P\left(z < \frac{31.5-27}{\sqrt{18.9}}\right)$	M1	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$=\Phi(1.035)$	M1	Appropriate area Φ, from standardisation formula P(*z*<…) in final solution, must be probability.
	= 0·850	A1	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	

284. 9709_s21_qp_51 Q: 2

A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

Answer:

Question	Answer	Marks	Guidance
	$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right)<z<\left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $=P\left(-\frac{0.5}{0.4}<z<\frac{0.5}{0.4}\right)$	M1	Use of ± Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
	$[=2\Phi(1.25)-1]$ $=2\times0.8944-1$	A1	For AWRT 0.8944 SOI
		M1	Appropriate area 2Φ – 1 OE, from final process, must be probability
	0.7888	A1	Accept AWRT 0.789
	Number of rods = 0.7888×500 = 394 or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
		5	

285. 9709_s21_qp_51 Q: 6

In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

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(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

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(c) Justify the use of your approximation in part **(b)**. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	$1 - P(10, 11, 12) = 1 - ({}^{12}C_{10}\, 0.6^{10}\, 0.4^{2} + {}^{12}C_{11}\, 0.6^{11}\, 0.4^{1} + {}^{12}C_{12}\, 0.6^{12}\, 0.4^{0})$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	$P(0,1,2,3,4,5,6,7,8,9) = {}^{12}C_0 0.6^0\, 0.4^{12} + {}^{12}C_1\, 0.6^1\, 0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9\, 0.6^9\, 0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

Question	Answer	Marks	Guidance
(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6 \times 150 \times 0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X < 81) = P\left(Z < \frac{80.5 - 90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

286. 9709_s21_qp_52 Q: 2

The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

Answer:

Question	Answer	Marks	Guidance
	$\left[P(X>1.1)=\frac{72}{2000}(=0.036)\right]$ $z=\pm1.798$	B1	$1.79 < z \leqslant 1.80$, $-1.80 \leqslant z < -1.79$ seen
	$\frac{1.1-1.04}{\sigma}=1.798$ $\left[\frac{0.06}{\sigma}=1.798\right]$	B1	1.1 and 1.04 substituted in ±standardisation formula, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Equate *their* ±standardisation formula to a z-value and to solve for the appropriate area leading to final answer (expect $\sigma < 0.5$). $\left(\text{Accept} \pm\frac{0.06}{\sigma}=z-\text{value}\right)$
	$\sigma=0.0334$	A1	$0.03335 \leq \sigma \leq 0.0334$. At least 3 3s.f.
		4	

287. 9709_s21_qp_52 Q: 5

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

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(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

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(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	$[(0.7)^3 =]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3 + {}^3C_1(0.15)^2(0.55) + {}^3C_2(0.15)(0.55)^2 + (0.55)^3 =]\ 0.343$	B1	Evaluated WWW
		1	
(b)	$1 - (0.85^9 + {}^9C_1\,0.15^1\,0.85^8 + {}^9C_2\,0.15^2\,0.85^7)$ $[1 - (0.231617 + 0.367862 + 0.259667)]$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\,0.15^3\,0.85^6 + {}^9C_4\,0.15^4\,0.85^5 + {}^9C_5\,0.15^5\,0.85^4 + {}^9C_6\,0.15^6\,0.85^3 +$ ${}^9C_7\,0.15^7\,0.85^2 + {}^9C_8\,0.15^8\,0.85 + 0.15^9$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	

Question	Answer	Marks	Guidance
(c)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. $(2.765 \le \sigma \le 2.77$ imply correct variance)
	$[(X\ge12)=]P\left(Z>\frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

288. 9709_s21_qp_53 Q: 5

The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. [5]

The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$z_1 = \frac{4-\mu}{\delta} = -1.378$	B1	$1.378 \leqslant z_1 \leq 1.379$ or $-1.379 \leqslant z_1 \leqslant -1.378$
	$z_2 = \frac{10-\mu}{\sigma} = 0.842$	B1	$0.841 \leqslant z_2 \leqslant 0.842$ or $-0.842 \leqslant z_2 \leqslant -0.841$
	Solve to find at least one unknown: $\frac{4-\mu}{\sigma} = -1.378$ $\frac{10-\mu}{\sigma} = 0.842$	M1	Use of ±standardisation formula once with μ, σ, a z-value and 4 or 10, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve two equations in μ and σ.
	$\sigma = 2.70$ $\mu = 7.72$	A1	$2.70 \leqslant \sigma \leqslant 2.71$ $7.72 \leqslant \mu \leqslant 7.73$
		5	
(b)	$\Phi(2) - \Phi(-2) = 2\Phi(2) - 1$	M1	Identifying 2 and –2 as the appropriate z-values
	$2 \times their\, 0.9772 - 1$	B1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.9544 or 0.9545	A1	Accept AWRT 0.954
	$0.9544 \times 800 = 763.52$ 763 or 764	B1 FT	FT *their* 4SF (or better) probability, final answer must be positive integer
		4	

289. 9709_s21_qp_53 Q: 7

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

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(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

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In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

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(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

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Answer:

Question	Answer	Marks	Guidance
(a)(i)	$\frac{40}{800}$ or $\frac{1}{20}$ or 0.05	B1	
		1	
(a)(ii)	$\frac{177}{223+177+40}$	M1	*Their* 223 + 177 + 40 seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
	Alternative method for Question 7(a)(ii)		
	$P(G\mid S)=\frac{P(G\cap S)}{P(S)}=\frac{\frac{177}{800}}{\frac{223+177+40}{800}}=\frac{\frac{177}{800}}{\frac{440}{800}}=\frac{\frac{177}{800}}{\frac{11}{20}\text{ or }0.55}$	M1	*Their* P(*S*) seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
		2	
7(b)(i)	P(0, 1, 2) = $^{10}C_0(0.35)^0(0.65)^{10}+{}^{10}C_1(0.35)^1(0.65)^9+{}^{10}C_2(0.35)^2(0.65)^8$	M1	One term: $^{10}C_x p^x(1-p)^{10-x}$ for $0 < x < 10$, any $0<p<1$
	0.013463 + 0.072492 + 0.17565	A1	Correct unsimplified expression, or better
	0.262	A1	
		3	

Question	Answer	Marks	Guidance
(b)(ii)	Mean = $120\times0.35[=42]$ Variance = $120\times0.35\times0.65[=27.3]$	B1	Correct mean and variance seen, allow unsimplified
	$P(X>32)=P(Z>\frac{32.5-42}{\sqrt{27.3}})=P(Z>-1.818)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number), condone σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 31.5 or 32.5
	$\Phi(1.818)$	M1	Appropriate area Φ, from final process, must be probability
	0.966	A1	$0.965 \leqslant p \leqslant 0.966$
		5	

290. 9709_w21_qp_51 Q: 7

The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) **(i)** On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

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(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

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(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	$P(X > 142) = P\left(Z > \frac{142-125}{24}\right)$	M1	Substitution of correct values into the ±Standardisation formula, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$[= P(Z > 0.7083) =] 1 - 0.7604$	M1	Appropriate numerical area Φ, from final process, must be probability, expect $p < 0.5$.
	0.2396	A1	$0.239 \leqslant p \leqslant 0.240$ to at least 3sf.
	Their 0.2396×365 [= 87.454]	M1	**FT** *their* 4sf (or better) probability.
	87 or 88	A1 FT	Final answer must be positive integer, no indication of approximation/rounding, only dependent on previous **M** mark. **SC B1 FT** for *their* 3sf probability × 365 = integer value, condone 0.24 used.
		5	
(a)(ii)	$P(0, 1) = 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$ [= 0.064628 + 0.20364]	M1	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1 FT	Correct unsimplified expression using *their* probability to at least 3sf from **(a)(i)** or correct.
	0.268	A1	AWRT, WWW.
		3	
(b)	$z = \pm 1.282$	B1	Correct value only, critical value.
	$\frac{t-125}{24} = -1.282$	M1	Use of ± Standardisation formula with correct values substituted, allow continuity correction, σ^2, $\sqrt{\sigma}$, to form an equation with a z-value and not probability.
	$t = 94.2$	A1	AWRT, condone AWRT 94.3. Not dependent on **B** mark.
		3	

291. 9709_w21_qp_52 Q: 6

The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

(a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. [3]

(b) 20% of employees take longer than t minutes to complete the task.

Find the value of t. [3]

(c) Find the probability that the time taken to complete the task by a randomly chosen employee differs from the mean by less than 15.0 minutes. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\left[P(X > 28.6) =\right] P\left(Z > \frac{28.6 - 32.2}{9.6}\right)$ $\left[= P(Z > -0.375)\right]$	M1	28.6, 32.2 and 9.6 substituted appropriately in ± Standardisation formula once, allow continuity correction of ± 0.05, no σ^2, $\sqrt{\sigma}$.
	$[\Phi(\textit{their}\ 0.375) =]$ *their* 0.6462	M1	Appropriate numerical area, from final process, must be probability, expect > 0.5.
	0.646	A1	AWRT
		3	
(b)	$z = \pm 0.842$	B1	$0.841 < z \leqslant 0.842$ or $-0.842 \leqslant z < -0.841$ seen.
	$\frac{t - 32.2}{9.6} = 0.842$	M1	Substituting 32.2 and 9.6 into ± standardisation formula, no continuity correction, allow σ^2, $\sqrt{\sigma}$, must be equated to a z-value.
	$t = 40.3$	A1	$40.28 \leqslant t \leqslant 40.3$ WWW
		3	

Question	Answer	Marks	Guidance
(c)	$P\left(-\frac{15}{9.6} < Z < \frac{15}{9.6}\right)$ $P(-1.5625 < Z < 1.5625)$	M1	Identifying at least one of $\frac{15}{9.6}$ and $-\frac{15}{9.6}$ as the appropriate z-values or substituting *their* (32.2 ± 15) into ± Standardisation formula once, no continuity correction, σ^2 nor $\sqrt{\sigma}$. Condone ±1.563 for **M1**.
	$[2\ \Phi(\frac{15}{9.6}) - 1]$ $= 2 \times 0.9409 - 1$	A1	$p = 0.941$ AWRT SOI
		M1	Appropriate area $2\Phi - 1$ oe, (eg $1 - 2 \times 0.0591$, $2 \times (0.9409 - 0.5)$ or $0.9409 - 0.0591$), from final process, must be probability > 0.5.
	0.882	A1	
		4	

292. 9709_w21_qp_53 Q: 4

Raj wants to improve his fitness, so every day he goes for a run. The times, in minutes, of his runs have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the probability that on a randomly chosen day Raj runs for more than 43.2 minutes. [3]

(b) Find an estimate for the number of days in a year (365 days) on which Raj runs for less than 43.2 minutes. [2]

(c) On 95% of days, Raj runs for more than t minutes.

Find the value of t. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$P(X > 43.2) = P\left(Z > \frac{43.2 - 41.2}{3.6}\right) = P(Z > 0.5556)$	M1	Use of ±Standardisation formula once, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$1 - \Phi(0.5556) = 1 - 0.7108$	M1	Appropriate area Φ, from final process, must be probability.
	0.289	A1	AWRT
		3	
(b)	Probability = 1 – *their* **(a)** = 1 – 0.2892 = 0.7108	B1FT	1 – *their* **(a)** or correct.
	0.7108 × 365 = 259.4 259, 260	B1FT	FT *their* 4SF (or better) probability, final answer must be positive integer.
		2	
(c)	$z = \pm 1.645$	B1	CAO, critical z value.
	$\frac{t - 41.2}{3.6} = -1.645$	M1	Use of ±standardisation formula with μ, σ equated to a z-value, no continuity correction, allow σ^2, $\sqrt{\sigma}$.
	$t = 35.3$	A1	
		3	

293. 9709_m20_qp_52 Q: 3

The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of these apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. [3]

(b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$P(X > 87) = P\left(Z > \frac{87-82}{\sigma}\right) = 0.22$	M1	Using ± standardisation formula, not σ^2, not $\sqrt{\sigma}$, no continuity correction
	$P\left(Z < \frac{5}{\sigma}\right) = 0.78$ $\left(\frac{5}{\sigma} =\right) 0.772$	B1	AWRT ±0.772 seen B0 for ±0.228
	$\sigma = 6.48$	A1	
		3	
(b)	$P\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right) = P(-0.6176 < Z < 0.6176)$	M1	Using ±4 used within a standardisation formula (SOI), allow σ^2, $\sqrt{\sigma}$ and continuity correction
		M1	Standardisation formula applied to **both** *their* ±4
	$\Phi = 0.7317$ Prob = $2\Phi - 1 = 2(0.7317) - 1$	M1	Correct area $2\Phi - 1$ oe linked to final solution
	$= 0.463$	A1	
		4	

294. 9709_m20_qp_52 Q: 5

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	1 – P(6, 7, 8) $= 1 - ({}^8C_6\, 0.7^6 0.3^2 + {}^8C_7\, 0.7^7 0.3^1 + 0.7^8)$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x},\quad 0<p<1,\ x\neq 0$
	= 1 – 0.55177	A1	Correct unsimplified expression, or better
	= 0.448	A1	
	Alternative method for question 5(a)		
	P(0, 1, 2, 3, 4, 5) $= 0.3^8 + {}^8C_1\, 0.7^1 0.3^7 + {}^8C_2\, 0.7^2 0.3^6 + {}^8C_3\, 0.7^3 0.3^5 + {}^8C_4 0.7^4 0.3^4 + {}^8C_5\, 0.7^5 0.3^3$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x},\quad 0<p<1,\ x\neq 0$
		A1	Correct unsimplified expression, or better
	= 0.448	A1	
		3	
(b)	Mean $= 120\times 0.7 = 84$ Var $= 120\times 0.7\times 0.3 = 25.2$	B1	Correct mean and variance, allow unsimplified
	P(more than 75) $= \mathrm{P}\left(z > \frac{75.5-84}{\sqrt{25.2}}\right)$	M1	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
		M1	Using continuity correction 75.5 or 74.5
	P($z > -1.693$)	M1	Appropriate area Φ , from final process, must be a probability
	= 0.955	A1	Allow $0.9545 < p \leqslant 0.955$
		5	

295. 9709_s20_qp_51 Q: 6

The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

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The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

Answer:

Question	Answer	Marks
(a)	$\text{P}\left(\frac{50-54}{6.1} < z < \frac{60-54}{6.1}\right) = \text{P}(-0.6557 < Z < 0.9836)$	M1
	Both values correct	A1
	$\Phi(0.9836) - \Phi(-0.6557) = \Phi(0.9836) + \Phi(0.6557) - 1$ $= 0.8375 + 0.7441 - 1$ (Correct area)	M1
	0.582	A1
		4

Question	Answer	Marks
(b)	$\frac{45-\mu}{\sigma} = -0.994$	B1
	$\frac{56-\mu}{\sigma} = 1.372$	B1
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	M1
	$11 = 2.366\,\sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	M1
	$\sigma = 4.65$, $\mu = 49.6$	A1
		5

296. 9709_s20_qp_52 Q: 4

Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. [3]

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places. [2]

(c) Find the height above which trees are classified as tall. [3]

Answer:

Question	Answer	Marks
(a)	$P(X < 25) = P\left(z < \frac{25-40}{12}\right) = P(z < -1.25) P(X < 25) = P(z <)$	M1
	$1 - 0.8944$	M1
	0.106	A1
		3
(b)	0.8944 divided by 3 (**M1** for 1 - *their* **(a)** divided by 3)	M1
	0.298 **AG**	A1
		2
(c)	0.2981 gives $z = 0.53$	B1
	$\frac{h-40}{12} = 0.53$	M1
	$h = 46.4$	A1
		3

297. 9709_s20_qp_52 Q: 7

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

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(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

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(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

Answer:

Question	Answer	Marks
(a)	$1 - P(10, 11, 12)$ $= 1 - [{}^{12}C_{10}\,0.72^{10}0.28^2 + {}^{12}C_{11}\,0.72^{11}0.28^1 + 0.72^{12}]$	M1
	$1 - (0.19372 + 0.09057 + 0.01941)$	A1
	0.696	A1
		3
(b)	$0.28^3 \times 0.72 = 0.0158$	B1
		1

Question	Answer	Marks
(c)	Mean = $100 \times 0.72 = 72$ Var = $100 \times 0.72 \times 0.28 = 20.16$	M1
	P(less than 64) = $P\left(z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their* μ *and* σ into ±standardisation formula with a numerical value for '*63.5*')	M1
	Using either 63.5 or 64.5 within a ±standardisation formula	M1
	Appropriate area Φ, from standardisation formula P(z<…) in final solution $= P(z < -1.893)$	M1
	0.0292	A1
		5

298. 9709_s20_qp_53 Q: 3

In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

Answer:

Question	Answer	Marks
(a)	$P(X < 21) = P\left(z < \frac{21 - 15.8}{4.2}\right) = \Phi(1.238)$	M1
	0.892	A1
		2
(b)	$z = \pm 0.674$	B1
	$\frac{k - 15.8}{4.2} = 0.674$	M1
	18.6	A1
		3

299. 9709_s20_qp_53 Q: 5

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

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(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

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(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

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On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

Answer:

Question	Answer	Marks
(a)	$\frac{1}{\frac{1}{4}} = 4$	B1
		1
(b)	$\frac{9}{64}$ $(= 0.141)$	B1
		1
(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (**FT** *their* probability/mean from part **(a)**)	M1
	0.763	A1
		2
(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	M1
	P(more than 25) $= P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$	M1
	$P(z > 1.42)$	M1
	$1 - 0.9222$	M1
	0.0778	A1
		5

300. 9709_w20_qp_51 Q: 5

The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

(b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t. [3]

(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

Answer:

(a)	$P(X > 4.2) = P(z > \frac{4.2-3.5}{0.9})$ $= P(z > 0.7778)$	M1	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0·7818	M1	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·218	A1	
		3	
(b)	$z = -1.282$	B1	±1.282 seen (critical value)
	$\frac{t-3.5}{0.9} = -1.282$	M1	An equation using ±standardisation formula with a *z*-value, condone $\sqrt{\sigma}$, σ^2 and continuity correction
	$t = 2.35$	A1	AWRT, only dependent on M mark
		3	

Question	Answer	Marks	Guidance
(c)	P(2.8 < X < 4.2) = 1 – 2 × *their* **5(a)** ≡ 2(1 – *their* **5(a)**) – 1 ≡ 2(0·5 – *their* **5(a)**) = 0·5636	B1 FT	FT from *their* **5(a)** < 0.5 or correct Accept unevaluated probability OE Accept 0·564
	Number of days = 365 × 0·5636 = 205·7	M1	365 × *their p*
	So, 205 (days)	A1 FT	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
	Alternative method for question 5(c)		
	$P\left(\frac{2.8-3.5}{0.9} < z < \frac{4.2-3.5}{0.9}\right)$ $= \Phi(0.7778) - (1 - \Phi 0.7778)$ = 0·7818 – (1 – 0·7818) = 0·5636	B1	$0{\cdot}5635 < p \leqslant 0{\cdot}564$ OE
	Number of days = 365 × 0·5636 = 205·7	M1	365 × *their p*
	So, 205 (days)	A1 FT	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
		3	

301. 9709_w20_qp_52 Q: 3

Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t. [3]

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(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$P(X > 11.3) = P(z > \frac{11.3-10.1}{1.3}) = P(z > 0\cdot 9231)$	M1	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0.822	M1	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·178	A1	0.1779…
		3	
(b)	$z = -0\cdot 674$	B1	±0.674 seen (critical value)
	$\frac{t-10.1}{1.3} = -0\cdot 674$	M1	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$ or σ^2, continuity correction.
	$t = 9\cdot 22$	A1	AWRT. Only dependent on M1
		3	

Question	Answer	Marks	Guidance
(c)	$P(8.9 < X < 11.3) = 1 - 2 \times$ *their* **3(a)** ≡ 2(1 – *their* **3(a)**) – 1 ≡ 2(0·5 – *their* **3(a)**) =0.644	B1 FT	FT from *their* **3(a)** < 0·5 or correct, accept unevaluated probability OE
	Number of days = 90 × 0·644 = 57·96	M1	90 × *their* p seen, $0 < p < 1$
	So 57 (days)	A1 FT	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
	Alternative method for question 3(c)		
	$P\left(\frac{8\cdot 9-10\cdot 1}{1\cdot 3} < z < \frac{11.3-10.1}{1\cdot 3}\right)$ $= \Phi(0\cdot 9231) - (1-\Phi(0\cdot 9231))$ oe = 0·822 – (1 – 0·822) = 0·644	B1	Accept unevaluated probability
	Number of days = 90 × 0·644 = 57·96	M1	90 × *their* p seen, $0 < p < 1$
	So 57 (days)	A1 FT	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
		3	

302. 9709_w20_qp_53 Q: 1

The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

(a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. [3]

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(b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$P(56 < X < 66) = P\left(\frac{56-62}{5} < z < \frac{66-62}{5}\right)$ $= P(-1.2 < z < 0.8)$	M1	Using ± standardisation formula at least once, no $\sqrt{\sigma}$ or σ^2, allow continuity correction
	$\Phi(0.8) + \Phi(1.2) - 1$ $= 0.7881 + 0.8849 - 1$	M1	Appropriate area Φ, from standardisation formula in final solution
	0.673	A1	
		3	
(b)	$z = 1.127$	B1	±(1.126 – 1.127) seen, 4 sf or more
	$\frac{60t-62}{5} = 1.127$ $60t = 5.635+62=67.635$	M1	z-value = $\pm\frac{(60t-62)}{5}$ condone z-value = $\pm\frac{(t-62)}{5}$ no continuity correction, condone $\sqrt{\sigma}$ or σ^2
	$t = 1.13$	A1	CAO
		3	

303. 9709_w20_qp_53 Q: 4

The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	$0.65^7 + {}^7C_1\ 0.65^6\ 0.35^1 + {}^7C_2\ 0.65^5\ 0.35^2$	M1	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1$, any p, $x \neq 0, 7$
	$0.049022 + 0.184776 + 0.29848$	A1	Correct unsimplified answer
	0.532	A1	
		3	
(b)	Mean $= 142 \times 0.35 = 49.7$ Variance $= 142 \times 0.35 \times 0.65 = 32.305$	B1	Correct unsimplified np and npq (condone $\sigma = 5.684$ evaluated)
	$P(X > 40) = P(z > \frac{40.5 - 49.7}{\sqrt{32.305}})$	M1	Substituting *their* μ and σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z > -1.619)$	M1	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z > \ldots)$ in final solution, must be probability
	0.947	A1	Correct final answer
		5	

304. 9709_m19_qp_62 Q: 3

The times taken, in minutes, for trains to travel between Alphaton and Beeton are normally distributed with mean 140 and standard deviation 12.

(i) Find the probability that a randomly chosen train will take less than 132 minutes to travel between Alphaton and Beeton. [3]

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(ii) The probability that a randomly chosen train takes more than k minutes to travel between Alphaton and Beeton is 0.675. Find the value of k. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$P(X< 132) = P\left(Z < \frac{132-140}{12}\right) = P(Z < -0.6667)$	**M1**	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	$= 1 - 0.7477$	**M1**	Appropriate area Φ from standardisation formula P(z<….) in final solution
	= 0.252 awrt	**A1**	Condone linear interpolation = 0.25243
		3	
(ii)	P(time>k) = 0.675, $z = -0.454$	**B1**	±0.454 seen
	$\frac{k-140}{12} = -0.454$	**M1**	An equation using the standardisation formula with a z-value (not $1 - z$), condone σ^2 or $\sqrt{\sigma}$
	$k = 135$, 134.6, 134.55	**A1**	B0M1A1 max from –0.45
		3	

305. 9709_m19_qp_62 Q: 6

The results of a survey by a large supermarket show that 35% of its customers shop online.

(i) Six customers are chosen at random. Find the probability that more than three of them shop online. [3]

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(ii) For a random sample of n customers, the probability that at least one of them shops online is greater than 0.95. Find the least possible value of n. [3]

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(iii) For a random sample of 100 customers, use a suitable approximating distribution to find the probability that more than 39 shop online. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(4, 5, 6) = {}^6C_4\, 0.35^4 0.65^2 + {}^6C_5\, 0.35^5 0.65^1 + 0.35^6$	M1	Binomial term of form ${}^6C_x p^x(1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
		A1	Correct unsimplified answer
	$= 0.117$	A1	
		3	
(ii)	$1 - 0.65^n > 0.95$ $0.65^n < 0.05$	M1	Equation or inequality involving '0.65^n or 0.35^n' and '0.95 or 0.05'
	$n > \dfrac{\log 0.05}{\log 0.65} = 6.95$	M1	Attempt to solve *their* exponential equation using logs or Trial and Error.
	$n = 7$	A1	CAO
		3	
(iii)	Mean $= 0.35 \times 100 = 35$ Variance $= 0.35 \times 0.65 \times 100 = 22.75$	B1	Correct unsimplified *np* and *npq*,
	$P\left(z > \dfrac{39.5 - 35}{\sqrt{22.75}}\right) = P(z > 0.943)$	M1	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for '*39.5*'.
		M1	Using continuity correction 39.5 or 40.5
	$= 1 - 0.8272$	M1	Appropriate area Φ from standardisation formula P(z>….) in final solution, (>0.5 if z is -ve, <0.5 if z is +ve)
	$= 0.173$	A1	Final answer
		5	

306. 9709_s19_qp_61 Q: 5

In a certain country the probability that a child owns a bicycle is 0.65.

(i) A random sample of 15 children from this country is chosen. Find the probability that more than 12 own a bicycle. [3]

(ii) A random sample of 250 children from this country is chosen. Use a suitable approximation to find the probability that fewer than 179 own a bicycle. [4]

Answer:

Question	Answer	Marks	Guidance
(i)	(P > 12) = P(13, 14, 15)	**M1**	Binomial term of form $^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$ any p, $x \neq 15,0$
	$= {}^{15}C_{13}(0.65)^{13}(0.35)^2 + {}^{15}C_{14}(0.65)^{14}(0.35)^1 + (0.65)^{15}$	**A1**	Correct unsimplified answer
	= 0.0617	**A1**	SC if use np and npq with justification give (12.5 – 9.75)/√3.41 M1 1–F(1.489) A1 0.0681 A0
		3	
(ii)	mean = 250 × 0.65 = 162.5 variance = 250 × 0.65 × 0.35 = 56.875	**B1**	Correct unsimplified *np* and *npq*
	$P(< 179) = P(z < \frac{178.5-162.5}{\sqrt{56.875}}) = P(z < 2.122)$	**M1**	Substituting *their* μ and σ (condone σ^2) into the Standardisation Formula with a numerical value for *'178.5'*. Continuity correct not required for this M1. Condone ± standardisation formula
	Using continuity correction 178.5 or 179.5	**M1**	
	= 0.983	**A1**	Correct final answer
		4	

307. 9709_s19_qp_61 Q: 7

The weight of adult female giraffes has a normal distribution with mean 830 kg and standard deviation 120 kg.

(i) There are 430 adult female giraffes in a particular game reserve. Find the number of these adult female giraffes which can be expected to weigh less than 700 kg. [4]

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(ii) Given that 90% of adult female giraffes weigh between $(830 - w)$ kg and $(830 + w)$ kg, find the value of w. [3]

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The weight of adult male giraffes has a normal distribution with mean 1190 kg and standard deviation σ kg.

(iii) Given that 83.4% of adult male giraffes weigh more than 950 kg, find the value of σ. [3]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(<700) = P\left(z < \frac{700-830}{120}\right) = P(z < -1.083)$	M1	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	$= 1 - 0.8606$	M1	Appropriate area Φ from standardisation formula P(z<….) in final probability solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	$= 0.1394$	A1	Correct final probability rounding to 0.139
	Expected number of female adults = 430 × *their* 0.1394 = 59.9 So 59 or 60	B1	**FT** *their* 3 or 4 SF probability, rounded or truncated to integer
		4	

Question	Answer	Marks	Guidance
(ii)	P(giraffe < 830+w) = 95% so z = 1.645	B1	±1.645 seen (critical value)
	$\frac{(830+w)-830}{120} = \frac{w}{120} = 1.645$	M1	An equation using the standardisation formula with a z-value (not $1 - z$), condone σ^2 or $\sqrt{\sigma}$ not 0.8519, 0.8289
	$w = 197$	A1	Correct answer
		3	
(iii)	P(male > 950) = 0.834, so $z = -0.97$	B1	± 0.97 seen
	$\frac{950-1190}{\sigma} = -0.97$	M1	Using ± standardisation formula, condone continuity correction, σ^2 or $\sqrt{\sigma}$, condone equating with non z-value not 0.834, 0.166
	$\sigma = 247$	A1	Condone $-\sigma = -247$. www.
		3	

308. 9709_s19_qp_62 Q: 2

The volume of ink in a certain type of ink cartridge has a normal distribution with mean 30 ml and standard deviation 1.5 ml. People in an office use a total of 8 cartridges of this ink per month. Find the expected number of cartridges per month that contain less than 28.9 ml of this ink. [4]

Answer:

Question	Answer	Marks	Guidance
	$P(<28.9) = P\left(z < \frac{28.9-30}{1.5}\right)$	B1	Using ± standardising formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$,
	$= P(z < -0.733)$ $= 1 - 0.7682$	M1	Appropriate area Φ from standardisation formula P(z <….) in final probability solution, Must be a probability, e.g. 1 – 0.622 is M0
	= 0.2318	A1	Correct final probability rounding to 0.232. (Only requires M1 not B1 to be awarded
	Number of cartridges is *their* 0.2318 × 8 = 1.85, so 2 (Also accept 1 but not both)	B1	**FT** using *their* 4 SF (or better) value, ans. rounded or truncated to integer, no approximation indicated.
		4	

309. 9709_s19_qp_62 Q: 4

It is known that 20% of male giant pandas in a certain area weigh more than 121 kg and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weights of male giant pandas in this area. [5]

Answer:

Question	Answer	Marks	Guidance
	$z = 0.842 = \left(\frac{121-\mu}{\sigma}\right)$ so $0.842\sigma = 121 - \mu$	**B1**	± 0.842 seen but B0 if 1 ± 0.842 oe seen
		M1	One appropriate standardisation equation with a z-value, μ, σ and 121 **or** 102, condone continuity correction. Not 0.158, 0.42,…
	$z = -0.58 = \left(\frac{102-\mu}{\sigma}\right)$ so $-0.58\sigma = 102 - \mu$	**B1**	± 0.58(0) seen but B0 if 1 ± 0.58 oe seen
	Solving	**M1**	Correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable, condone 1 numerical slip
	$\sigma = 13.4$ $\mu = 110$	**A1**	If M0A0 scored (i.e. no algebraic elimination seen), **SC B1** can be awarded for both answers correct Consistent use of σ^2 or $\sqrt{\sigma}$ throughout apply **MR** penalty to A mark or SC B mark.
		5	

310. 9709_s19_qp_63 Q: 1

The time taken, in minutes, by a ferry to cross a lake has a normal distribution with mean 85 and standard deviation 6.8.

(i) Find the probability that, on a randomly chosen occasion, the time taken by the ferry to cross the lake is between 79 and 91 minutes. [3]

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(ii) Over a long period it is found that 96% of ferry crossings take longer than a certain time t minutes. Find the value of t. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$P(79 < X < 91) = P\left(\frac{79-85}{6.8} < Z < \frac{91-85}{6.8}\right)$ $= P(-0.8824 < Z < 0.8824)$	M1	Using ± standardisation formula for either 79 or 91, no continuity correction
	$= \Phi(0.8824) - \Phi(-0.8824)$ $= 0.8111 - (1 - 0.8111)$	M1	Correct area (Φ – Φ) with one +ve and one –ve z-value or 2 Φ – 1 or 2(Φ – 0.5)
	$= 0.622$	A1	Correct answer
		3	
(ii)	$z = -1.751$	B1	± 1.751 seen
	$-1.751 = \frac{t-85}{6.8}$	M1	An equation using ± standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	$t = 73.1$	A1	Correct answer
		3	

311. 9709_s19_qp_63 Q: 5

On average, 34% of the people who go to a particular theatre are men.

(i) A random sample of 14 people who go to the theatre is chosen. Find the probability that at most 2 people are men. [3]

(ii) Use an approximation to find the probability that, in a random sample of 600 people who go to the theatre, fewer than 190 are men. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(0, 1, 2) = (0.66)^{14} + {}^{14}C_1(0.34)(0.66)^{13} + {}^{14}C_2(0.34)^2(0.66)^{12}$	**M1**	Binomial term of form ${}^{14}C_x p^x(1-p)^{14-x}$ $0 < p < 1$ any p, $x \neq 14,0$
	= 0.0029758 + 0.02146239 + 0.071866	**A1**	Correct unsimplified answer
	= 0.0963	**A1**	Correct answer
		3	
(ii)	Mean =600 × 0.34 = 204, Var = 600 × 0.34 × 0.66 = 134.64	**B1**	Correct unsimplified *np* and *npq* (or sd = 11.603 or Variance = 3366/25)
	$P(<190) = P\left(z < \frac{189.5-204}{\sqrt{134.64}}\right) = P(z < -1.2496)$	**M1**	Substituting *their* μ and σ, (no σ^2 or $\sqrt{\sigma}$) into the Standardisation Formula with a numerical value for *'189.5'*. Condone ± standardisation formula
		M1	Using continuity correction 189.5 or 190.5 within a Standardisation formula
	$= 1 - \Phi(1.2496)$	**M1**	Appropriate area Φ from standardisation formula P(z<….) in final solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	= 1 – 0.8944 = 0.106	**A1**	Correct final answer
		5	

312. 9709_w19_qp_61 Q: 7

The shortest time recorded by an athlete in a 400 m race is called their personal best (PB). The PBs of the athletes in a large athletics club are normally distributed with mean 49.2 seconds and standard deviation 2.8 seconds.

(i) Find the probability that a randomly chosen athlete from this club has a PB between 46 and 53 seconds. [4]

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(ii) It is found that 92% of athletes from this club have PBs of more than t seconds. Find the value of t. [3]

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Three athletes from the club are chosen at random.

(iii) Find the probability that exactly 2 have PBs of less than 46 seconds. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	$P(46 < X < 53) = P\left(\frac{46-49.2}{2.8} < Z < \frac{53-49.2}{2.8}\right)$	**M1**	Using ± standardisation formula for either 46 or 53, no continuity correction, σ^2 or $\sqrt{\sigma}$
	$P(-1.143 < Z < 1.357)$	**A1**	Both standardisations correct unsimplified
	$\Phi(1.357)+\Phi(1.143)-1$ $= 0.9126 + 0.8735 - 1$	**M1**	Correct final area
	0.786	**A1**	Final answer
		4	

Question	Answer	Marks	Guidance
(ii)	$\frac{t-49.2}{2.8} = -1.406$	**B1**	±1.406 seen
		M1	An equation using ± standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	45.3	**A1**	
		3	
(iii)	$P(X < 46) = 0.1265$	**M1**	Calculated or ft from **(i)**
	$P(2PB < 46) = 3(1-0.1265)0.1265^2$	**M1**	$3(1-p)p^2$, $0<p<1$
	0.0419	**A1**	
		3	

313. 9709_w19_qp_62 Q: 4

In Quarendon, 66% of households are satisfied with the speed of their wifi connection.

(i) Find the probability that, out of 10 households chosen at random in Quarendon, at least 8 are satisfied with the speed of their wifi connection. [3]

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(ii) A random sample of 150 households in Quarendon is chosen. Use a suitable approximation to find the probability that more than 84 are satisfied with the speed of their wifi connection. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	P(8, 9, 10) = $^{10}C_8\, 0.66^8\, 0.34^2 + {}^{10}C_9\, 0.66^9\, 0.34^1 + 0.66^{10}$	M1	Correct binomial term, $^{10}C_a\, 0.66^a(1–0.66)^b$ $a+b=10,\ 0<a,b<10$
		A1	Correct unsimplified expression
	0.284	B1	CAO
		3	

Question	Answer	Marks	Guidance
(ii)	$np = 0.66\times150 = 99$ $npq = 0.66\times(1-0.66)\times150 = 33.66$	B1	Accept evaluated or unsimplified μ, σ^2 numerical expressions, condone $\sigma=\sqrt{33.66}=5.8017$ or 5.802 CAO
	$P(X>84)=P\left(Z>\frac{84.5-99}{\sqrt{33.66}}\right)$	M1	$\pm$ Standardise, $\frac{x-\textit{their}\ 99}{\sqrt{\textit{their}\ 33.66}}$, condone σ^2, x a value
		M1	84.5 or 83.5 used in *their* standardisation formula
	$(=P(Z>-2.499))$	M1	Correct final area
	0.994	A1	Final answer (accept 0.9938) SC if no standardisation formula seen, B2 $P(Z>-2.499)=0.994$
		5	

314. 9709_w19_qp_62 Q: 6

The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]

(ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

(iii) Find the mean and standard deviation of the heights of trees of this type. [5]

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Answer:

Question	Answer	Marks	Guidance
(i)	$P(X<45)=P\left(Z<\frac{45-40}{8}\right)$ $=P(Z<0.625)$	M1	± Standardise, no continuity correction, σ^2 or $\sqrt{\sigma}$, formula must be seen
	0.734(0)	A1	CAO
		2	
(ii)	$1-2(1-(i))=2(i)-1=2((i)-0.5)$	M1	Use result of **part (i)** or recalculated to find area OE
	0.468	A1ft	0 < FT from **(i)** < 1 or correct.
		2	
(iii)	$P(X<10)=48/500=0.096$ $z=-1.305$	B1	$z=\pm 1.305$
	$P(X>24)=76/500=0.152$ $z=1.028$	B1	$z=\pm 1.028$
	$10-\mu=-1.305\sigma$ $24-\mu=1.028\sigma$	M1	Form 1 equation using 10 or 24 with μ, σ, z–value. Allow continuity correction, not $\sigma^2, \sqrt{\sigma}$
	$14=2.333\sigma$	M1	OE Solve two equations in σ and μ to form equation in one variable
	$\sigma=6.[00]$, $\mu=17.8[3]$	A1	CAO, WWW
		5	

315. 9709_w19_qp_63 Q: 4

The heights of students at the Mainland college are normally distributed with mean 148 cm and standard deviation 8 cm.

(i) The probability that a Mainland student chosen at random has a height less than h cm is 0.67. Find the value of h. [3]

120 Mainland students are chosen at random.

(ii) Find the number of these students that would be expected to have a height within half a standard deviation of the mean. [4]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(h < 148) = 0.67$	B1	$z = \pm 0.44$ seen
	$\frac{h-148}{8} = 0.44$	M1	z-value $= \pm\frac{(h-148)}{8}$
	$151.52 \approx 152$	A1	CAO
		3	
(ii)	$P(144 < X < 152) = P\left(\frac{144-148}{8} < Z < \frac{152-148}{8}\right)$	M1	Using ± standardisation formula for either 144 or 152, $\mu = 148$, $\sigma = 8$ and no continuity correction, allow σ^2 or $\sqrt{\sigma}$
	$= P\left(-\frac{1}{2} < Z < \frac{1}{2}\right) = 0.6915 - (1 - 0.6915) = 2 \times 0.6915 - 1$	M1	Correct final area legitimately obtained from phi(*their* z_2) – phi(*their* z_1)
	$= 0.383$	A1	Final probability answer
	$0.383 \times 120 = 45.96$ Accept 45 or 46 only	B1FT	*Their* prob (to 3 or 4 sf) × 120, rounded to a whole number or truncated
		4	

316. 9709_w19_qp_63 Q: 7

A competition is taking place between two choirs, the Notes and the Classics. There is a large audience for the competition.

- 30% of the audience are Notes supporters.
- 45% of the audience are Classics supporters.
- The rest of the audience are not supporters of either of these choirs.
- No one in the audience supports both of these choirs.

(i) A random sample of 6 people is chosen from the audience.

(a) Find the probability that no more than 2 of the 6 people are Notes supporters. [3]

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(b) Find the probability that none of the 6 people support either of these choirs. [2]

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(ii) A random sample of 240 people is chosen from the audience. Use a suitable approximation to find the probability that fewer than 50 do not support either of the choirs. [5]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)(a)	P(0, 1, 2) = $^6C_0\, 0.3^0\, 0.7^6 + {}^6C_1\, 0.3^1\, 0.7^5 + {}^6C_2\, 0.3^2\, 0.7^4$	M1	Binomial term of form $^6C_x p^x(1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
	0.1176 ... + 0.3025 ... + 0.3241 ...	A1	Correct unsimplified answer
	0.744	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
(i)(b)	P(support neither choir) = 1 – (0.3 + 0.45) = 0.25	M1	0.25^n seen alone, $1 < n \leqslant 6$
	P(6 support neither choir) = 0.25^6 = 0.000244 or $\frac{1}{4096}$	A1	Correct final answer
		2	
(ii)	Mean = 240 × 0.25 = 60 Variance = 240 × 0.25 × 0.75 = 45	B1FT	Correct unsimplified *240p* and *240pq* where p =*their* P(support neither choir) or 0.25
	$P(X < 50) = P\left(Z < \frac{49.5 - 60}{\sqrt{45}}\right) = P(Z < -1.565)$	M1	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'49.5'*.
		M1	Using continuity correction 49.5 or 50.5 within a standardisation expression
	1 – 0.9412	M1	Appropriate area Φ from standardisation formula P(z< …) in final solution, (< 0.5 if z is –ve, > 0.5 if z is +ve)
	0.0588	A1	Correct final answer
		5	

317. 9709_m18_qp_62 Q: 7

The weights of packets of a certain type of biscuit are normally distributed with mean 400 grams and standard deviation σ grams.

(i) In a random sample of 6000 packets of this type of biscuit, 225 packets weighed more than 410 grams. Find the value of σ. [4]

(ii) In a random sample of 500 packets of this type of biscuit, how many packets would you expect to find with weights that are more than 1.5 standard deviations from the mean? [4]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(X > 410) = 225/6000 = 0.0375$ $P\left(Z > \frac{410-400}{\sigma}\right) = 0.0375$: 0.9625	M1	Use $1 - 225/6000 = 0.9625$ to find z value
	z value = ±1.78	A1	z value: ± 1.78
	$\frac{10}{\sigma} = 1.78$	M1	$(410\text{-}400)/\sigma$ = *their* z (must be a z value)
	$\sigma = 5.62$	A1	
		4	
(ii)	We need $P(Z < -1.5)$ and $P(Z > 1.5)$	M1	Attempt at $P(Z < -1.5)$ or $P(Z > 1.5)$ $1 - \Phi(1.5)$ seen
	$\Phi(-1.5) + 1 - \Phi(1.5)$ $= 2 - 2\Phi(1.5)$	M1	Or equivalent expression with values
	$=2 - 2 \times 0.9332 = 0.1336$ (0.134)	A1	Correct to 3sf
	Number expected = 500×0.1336 = 66.8: 66 or 67 packets	B1ft	0.1336 used or FT their 4sf probability times 500, (not 0.9625 or 0.0375) rounded or truncated
		4	

318. 9709_m18_qp_62 Q: 8

The results of a survey at a certain large college show that the proportion of students who own a car is $\frac{1}{4}$.

(i) Five students at the college are chosen at random. Find the probability that at least four of these students own a car. [3]

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(ii) For a random sample of n students at the college, the probability that at least one of the students owns a car is greater than 0.995. Find the least possible value of n. [3]

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(iii) For a random sample of 160 students at the college, use a suitable approximate distribution to find the probability that fewer than 50 own a car. [4]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	$P(4) + P(5) = {}^5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$	M1	One binomial term, with $p < 1$, n=5, $p + q$=1
	= 0.014648.. + 0.00097656..	M1	Add 2 correct unsimplified binomial terms
	= 0.0156 or $\frac{1}{64}$	A1	
		3	
(ii)	$1 - P(0) > 0.995$: $0.75^n < 0.005$	M1	Equation or inequality involving 0.75^n and 0.005 or 0.25^n and 0.995
	$n\log 0.75 < \log 0.005$ $n > 18.4$:	M1	Attempt to solve *their* exponential equation using logs, or trial and error May be implied by their answer
	$n = 19$	A1	
		3	
(iii)	$p = 0.25$, $n = 160$: mean = 160 x 0.25 (= 40) variance = 160 x 0.25 x 0.75 (=30)	B1	Correct unsimplified mean and variance
	$P(X < 50) = P\left(Z < \frac{49.5 - 40}{\sqrt{30}}\right)$	M1	Use standardisation formulae must include square root.
		M1	Use continuity correction ±0.5 (49.5 or 50.5)
	$= P(Z < 1.734) = 0.959$	A1	Correct final answer
		4	

319. 9709_s18_qp_61 Q: 4

(a) The distance that car tyres of a certain make can travel before they need to be replaced has a normal distribution. A survey of a large number of these tyres found that the probability of this distance being more than 36 800 km is 0.0082 and the probability of this distance being more than 31 000 km is 0.6915. Find the mean and standard deviation of the distribution. [5]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$, where $3\sigma = 4\mu$ and $\mu \neq 0$. Find $P(X < 3\mu)$. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$z_1 = 2.4$	B1	± 2.4 seen accept 2.396
	$z_2 = -0.5$	B1	± 0.5 seen
	$2.4 = \frac{36800-\mu}{\sigma}$	M1	Either standardisation eqn with z value, not 0.5082, 0.7565, 0.0082, 0.6915, 0.3085, 0.6209, 0.0032 or any other probability
	$-0.5 = \frac{31000-\mu}{\sigma}$	M1	Sensible attempt to eliminate μ or σ by substitution or subtraction from their 2 equations (z-value not required), need at least 1 value stated
	σ = 2000 μ = 32000	A1	Both correct answers
		5	
(b)	$P(X < 3\mu) = P\left(z < \frac{3\mu-\mu}{(4\mu/3)}\right)$ or $P = \left(z < \frac{(9\sigma/4)-(3\sigma/4)}{\sigma}\right)$	M1	Standardise, in terms of one variable, accept σ^2 or $\sqrt{\sigma}$
	$P(z < \frac{6}{4})$	M1	$\frac{6}{4}$ or $\frac{6}{4\sigma}$ seen
	= 0.933	A1	Correct final answer
		3	

320. 9709_s18_qp_61 Q: 5

In Pelmerdon 22% of families own a dishwasher.

(i) Find the probability that, of 15 families chosen at random from Pelmerdon, between 4 and 6 inclusive own a dishwasher. [3]

(ii) A random sample of 145 families from Pelmerdon is chosen. Use a suitable approximation to find the probability that more than 26 families own a dishwasher. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	P(4, 5, 6) = $^{15}C_4(0.22)^4(0.78)^{11} + {}^{15}C_5(0.22)^5(0.78)^{10}$ +	M1	One binomial term $^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$
	$^{15}C_6(0.22)^6(0.78)^9$	A1	Correct unsimplified expression
	= 0.398	A1	Correct answer
		3	
(ii)	$\mu = 145 \times 0.22 = 31.9$ $\sigma^2 = 145 \times 0.22 \times 0.78 = 24.882$	B1	Correct unsimplified mean and variance
	$P(x > 26) = P\left(z > \frac{26.5 - 31.9}{\sqrt{24.882}}\right) = P(z > -1.08255)$	M1	Standardising must have sq rt
		M1	25.5 or 26.5 seen as a cc
	= $\Phi(1.08255)$	M1	Correct area Φ, must agree with their μ
	= 0.861	A1	Correct final answer accept 0.861, or 0.860 from 0.8604 not from 0.8599
		5	

321. 9709_s18_qp_62 Q: 3

(i) The volume of soup in Super Soup cartons has a normal distribution with mean μ millilitres and standard deviation 9 millilitres. Tests have shown that 10% of cartons contain less than 440 millilitres of soup. Find the value of μ. [3]

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(ii) A food retailer orders 150 Super Soup cartons. Calculate the number of these cartons for which you would expect the volume of soup to be more than 1.8 standard deviations above the mean. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$z = -1.282$	**B1**	± 1.282 seen
	$-1.282 = \frac{440-\mu}{9}$	**M1**	$\pm$Standardisation equation with 440, 9 and μ, equated to a z-value, (not $1 - z$-value or probability e.g. 0.1841, 0.5398, 0.6202, 0.8159)
	$\mu = 452$	**A1**	Correct answer rounding to 452, not dependent on B1
		3	
(ii)	$P(z > 1.8) = 1 - 0.9641 = 0.0359$	**B1**	
	Number $= 0.0359 \times 150$ $= 5.385$	**M1**	$p \times 150$, $0 < p < 1$
	(Number of cartons =) 5	**A1FT**	Accept either 5 or 6, not indicated as an approximation, e.g. ~, about **FT** *their* $p \times 150$, answer as an integer
		3	

322. 9709_s18_qp_62 Q: 7

In a certain country, 60% of mobile phones sold are made by Company A, 35% are made by Company B and 5% are made by other companies.

(i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company A. [3]

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(ii) Use a suitable approximation to find the probability that, out of a random sample of 130 people who buy a mobile phone, at least 50 choose a mobile phone made by Company B. [5]

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(iii) A random sample of n mobile phones sold is chosen. The probability that at least one of these phones is made by Company B is more than 0.98. Find the least possible value of n. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	**Method 1** P(< 11) = 1 – P(11, 12, 13)	M1	Binomial expression of form ${}^{13}C_x\ (p)^x(1-p)^{13-x}$, $0 < x < 13$, $0 < p < 1$
	$= 1 - {}^{13}C_{11}(0.6)^{11}(0.4)^2 - {}^{13}C_{12}(0.6)^{12}(0.4) - (0.6)^{13}$	M1	Correct unsimplified answer
	= 0.942	A1	CAO
	Method 2 P(< 11) = P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	M1	Binomial expression of form ${}^{13}C_x\ (p)^x(1-p)^{13-x}$ $0 < x < 13$, $0 < p < 1$
	$= (0.4)^{13} + {}^{13}C_1(0.4)^{12}(0.6) + \ldots + {}^{13}C_{10}(0.4)^3(0.6)^{10}$	M1	Correct unsimplified answer
	= 0.942	A1	CAO
		3	
(ii)	$\mu = 130 \times 0.35 = 45.5$ var $= 130 \times 0.35 \times 0.65 = 29.575$	B1	Correct unsimplified mean and var (condone $\sigma^2 = 29.6$, $\sigma = 5.438$)
	$P(\geqslant 50) = P\left(z > \frac{49.5 - 45.5}{\sqrt{29.575}}\right) = P(z > 0.7355)$	M1	Standardising, using $\pm\left(\frac{x - \textit{their}\ \text{mean}}{\textit{their}\ \sigma}\right)$, x = value to standardise 49.5 or 50.5 seen in ± standardisation equation
	$= 1 - \Phi(0.7355)$	M1	Correct final area
	= 1 – 0.7691	M1	
	= 0.231	A1	Correct final answer
		5	

Question	Answer	Marks	Guidance
(iii)	$1 - (0.65)^n > 0.98$ or $0.02 > (0.65)^n$	M1	Eqn or inequality involving, 0.65^n and 0.02 **or** 0.35^n and 0.98
	$n > 9.08$	M1	Attempt to solve their eqn or inequality by logs **or** trial and error
	$n = 10$	A1	CAO
		3	

323. 9709_s18_qp_63 Q: 6

The diameters of apples in an orchard have a normal distribution with mean 5.7 cm and standard deviation 0.8 cm. Apples with diameters between 4.1 cm and 5 cm can be used as toffee apples.

(i) Find the probability that an apple selected at random can be used as a toffee apple. [3]

(ii) 250 apples are chosen at random. Use a suitable approximation to find the probability that fewer than 50 can be used as toffee apples. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	$z_1 = \pm\frac{4.1-5.7}{0.8} = -2$ $\quad z_2 = \pm\frac{5-5.7}{0.8} = -0.875$	**M1**	At least one standardising no cc no sq rt no sq using 5.7 and 0.8 and either 4.1 or 5
	P(Toffee Apple) $= P(d < 5.0) - P(d < 4.1)$ $= P(z < -0.875) - P(z < -2)$ $= \Phi(-0.875) - \Phi(-2)$ $= \Phi(2) - \Phi(0.875)$	**M1**	Correct area $\Phi - \Phi$ legitimately obtained – need 2 negative z-values or 2 positives – not one of each
	= 0.9772 – 0.8092 = 0.168 (or 0.1908 – 0.0228)	**A1**	Correct final answer
	Total:	**3**	
(ii)	np = 250 × 0.168 = 42, npq = 34.944	**B1ft**	Correct unsimplified mean and var – ft their prob for **(i)** providing $(0 < p < 1)$ Implied by $\sigma = \sqrt{34.944} = 5.911$
	$P(< 50) = P\left(z < \frac{49.5-42}{\sqrt{34.944}}\right) = P(z < 1.2687)$	**M1**	± Standardising using 50, their mean and sd; must have sq rt.
		M1	49.5 or 50.5 seen as a cc
	$= \Phi(1.2687)$	**M1**	Correct area Φ(> 0.5 for + z and < 0.5 for –z)in their final answer
	= 0.898	**A1**	Correct final answer
	Total:	**5**	

324. 9709_w18_qp_61 Q: 4

(a) It is given that $X \sim N(31.4, 3.6)$. Find the probability that a randomly chosen value of X is less than 29.4. [3]

(b) The lengths of fish of a particular species are modelled by a normal distribution. A scientist measures the lengths of 400 randomly chosen fish of this species. He finds that 42 fish are less than 12 cm long and 58 are more than 19 cm long. Find estimates for the mean and standard deviation of the lengths of fish of this species. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	$P(X < 29.4) = P(Z < \frac{29.4 - 31.4}{\sqrt{3.6}})$ $= P(Z < -1.0541)$	**M1**	Standardise, no cc, must have sq rt.
	$= 1 - 0.8540$	**M1**	Obtain 1 − prob
	$= 0.146$	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
(b)	$P(X < 12) = \frac{42}{400} = 0.105$ and $P(X > 19) = \frac{58}{400} = 0.145$	**M1**	Eqn with μ, σ and a z-value. Allow cc, wrong sign, but not $\sqrt{\sigma}$ or σ^2
	$\frac{12 - \mu}{\sigma} = -1.253$	**B1**	Any form with z value rounding to ±1.25
	$\frac{19 - \mu}{\sigma} = 1.058$	**B1**	Any form with z value rounding to ±1.06
	$12 - \mu = -1.253\sigma$ $19 - \mu = 1.058\sigma$ $7 = 2.307\sigma$ or $36.455 + 2.307\mu = 0$ oe	**M1**	Solve 2 equations in μ, σ eliminating to 1 unknown
	$\mu = 15.8$, $\sigma = 3.03$	**A1**	Correct answers
		5	

325. 9709_w18_qp_61 Q: 5

At the Nonland Business College, all students sit an accountancy examination at the end of their first year of study. On average, 80% of the students pass this examination.

(i) A random sample of 9 students who will take this examination is chosen. Find the probability that at most 6 of these students will pass the examination. [3]

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(ii) A random sample of 200 students who will take this examination is chosen. Use a suitable approximate distribution to find the probability that more than 166 of them will pass the examination. [5]

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(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

Answer:

Question	Answer	Marks	Guidance
(i)	1 – (P(7) + P(8) + P(9)) $= 1 - ({}^9C_7\ 0.8^7 \times 0.2^2 + {}^9C_8\ 0.8^8 \times 0.2^1 + {}^9C_9\ 0.8^9 \times 0.2^0)$	**M1**	Any binomial term of form ${}^9C_x p^x (1-p)^{9-x}$, $x \neq 0$
		M1	Correct unsimplified expression
	= 1 – (0.3019899 + 0.3019899 + 0.1342177) = 0.262	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
(ii)	Mean = 200 × 0.8 = 160: var = 200 × 0.8 × 0.2 = 32	**B1**	Both unsimplified
	$P(X > 166) = P(Z > \frac{166.5 - 160}{\sqrt{32}})$	**M1**	Standardise, $z = \pm\frac{x - \textit{their}\,160}{\sqrt{\textit{their}\,32}}$ with square root
		M1	166.5 or 165.5 seen in attempted standardisation expression
	= P(Z > 1.149) = 1 – 0.8747	**M1**	1 – a Φ -value, correct area expression, linked to final answer
	= 0.125	**A1**	Correct final answer
		5	
(iii)	*np* = 160, *nq* = 40: both > 5 (so normal approx. holds)	**B1**	Both parts required
		1	

326. 9709_w18_qp_62 Q: 7

(a) The time, X hours, for which students use a games machine in any given day has a normal distribution with mean 3.24 hours and standard deviation 0.96 hours.

(i) On how many days of the year (365 days) would you expect a randomly chosen student to use a games machine for less than 4 hours? [3]

(ii) Find the value of k such that $P(X > k) = 0.2$. [3]

(iii) Find the probability that the number of hours for which a randomly chosen student uses a games machine in a day is within 1.5 standard deviations of the mean. [3]

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(b) The variable Y is normally distributed with mean μ and standard deviation σ, where $4\sigma = 3\mu$ and $\mu \neq 0$. Find the probability that a randomly chosen value of Y is positive. [3]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(a)(i)	$P(X<4)=P\left(Z<\frac{4-3.24}{0.96}\right)$	M1	±Standardisation formula, no cc, no sq rt, no square
	$= P\ (Z<0.7917)\ =\ 0.7858$	A1	$0.7855 < p \leqslant 0.7858$ or p = 0.786 Cao (implies M1A1 awarded), may be seen used in calculation
	their 0.7858 × 365 = 286 (or 287)	B1ft	*Their* probability × 365 provided 4sf probability **seen**. FT answer rounded or truncated to nearest integer. No approximation notation used.
		3	
(a)(ii)	$P(X<k)=P(Z<\frac{k-3.24}{0.96})=0.8$	B1	(z=) ± 0.842 seen
	$\frac{k-3.24}{0.96}=0.842$	M1	$z=\pm\frac{k-3.24}{0.96}$, allow cc, sq rt or square equated to a z-value (0.7881, 0.2119, 0.158, 0.8, 0.2 etc. are not acceptable)
	$k=4.05$	A1	Correct final answer, www
		3	
(a)(iii)	$P(-1.5<Z<1.5)=$	M1	$\Phi(z=1.5)$ or $\Phi(z=-1.5)$ seen used or $p=0.9332$ seen
	$\Phi(1.5)-\Phi(-1.5)=2\Phi(1.5)-1$ = 2 × 0.9332 – 1 oe	M1	Correct final area expression using *their* probabilities
	= 0.866	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
(b)	$P(Y>0)=P\left(Z>\frac{0-\mu}{\sigma}\right)\equiv P\left(Z>\frac{0-\mu}{3\mu/4}\right)$ or $P\left(Z>\frac{0-\left(\frac{4\sigma}{3}\right)}{\sigma}\right)$	M1	±Standardisation attempt in terms of one variable no sq rt or square, condone ±0.5 as cc
	$= P(Z>-4/3)$	A1	Correct unsimplified standardisation, no variables
	= 0.909	A1	Correct final answer
		3	

327. 9709_w18_qp_63 Q: 5

The weights of apples sold by a store can be modelled by a normal distribution with mean 120 grams and standard deviation 24 grams. Apples weighing less than 90 grams are graded as 'small'; apples weighing more than 140 grams are graded as 'large'; the remainder are graded as 'medium'.

(i) Show that the probability that an apple chosen at random is graded as medium is 0.692, correct to 3 significant figures. [4]

(ii) Four apples are chosen at random. Find the probability that at least two are graded as medium. [4]

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Answer:

Question	Answer	Marks	Guidance
(i)	$z_1 = \pm\frac{90-120}{24} = -\frac{5}{4}$, $z_2 = \pm\frac{140-120}{24} = \frac{5}{6}$	M1	At least one standardisation, no cc, no sq rt, no sq using 120 and 24 and either 90 or 140
	$= \Phi\left(\frac{20}{24}\right) - \Phi\left(-\frac{30}{24}\right)$	A1	–5/4 and 5/6 unsimplified
	$= \Phi(0.8333) - (1 - \Phi(1.25))$ = 0.7975 – (1 – 0.8944) or 0.8944 – 0.2025 = 0.6919	M1	Correct area Φ – Φ legitimately obtained and evaluated from phi(their z_2) – phi (their z_1)
	= 0.692 AG	A1	Correct answer obtained from 0.7975 and 0.1056 oe to 4sf or 0.6919 seen www
		4	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	Probability = P(2, 3, 4) $= 0.692^2(1-0.692)^2 \times {}^4C_2 + 0.692^3(1-0.692) \times {}^4C_3 + 0.692^4$	M1	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x \neq 0$ or 4
		B1	One correct bin term with $n = 4$ and $p = 0.692$.
	= 0.27256 + 0.40825 + 0.22931	M1	Correct unsimplified expression using 0.692 or better
	= 0.910	A1	Correct answer
	Method 2:		
	1 – P(0, 1) =	M1	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x \neq 0$ or 4
	$1 - 0.692^0(1-0.692)^4 \times {}^4C_0 - 0.692^1(1-0.692)^3 \times {}^4C_1$	B1	One correct bin term with $n = 4$ and $p = 0.692$
	= 1 – 0.00899 – 0.0808757	M1	Correct unsimplified expression using 0.692 or better
	= 0.910	A1	Correct answer
		4	

328. 9709_w18_qp_63 Q: 6

The lifetimes, in hours, of a particular type of light bulb are normally distributed with mean 2000 hours and standard deviation σ hours. The probability that a randomly chosen light bulb of this type has a lifetime of more than 1800 hours is 0.96.

(i) Find the value of σ. [3]

New technology has resulted in a new type of light bulb. It is found that on average one in five of these new light bulbs has a lifetime of more than 2500 hours.

(ii) For a random selection of 300 of these new light bulbs, use a suitable approximate distribution to find the probability that fewer than 70 have a lifetime of more than 2500 hours. [4]

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(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

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Answer:

Question	Answer	Marks	Guidance
(i)	P(X>1800) = 0.96, so P($Z > \frac{1800-2000}{\sigma}$) = 0.96	B1	± 1.75 seen
	$\Phi(\frac{200}{\sigma}) = 0.96$ $\frac{200}{\sigma} = 1.751$	M1	$z = \pm\frac{1800-2000}{\sigma}$, allow cc, allow sq rt, allow sq equated to a z-value
	$\sigma = 114$	A1	Correct final answer www
		3	
(ii)	Mean = 300 × 0.2 = 60 and variance = 300 × 0.2 × 0.8 = 48	B1	Correct unsimplified mean and variance
	P(X< 70) = P($Z > \frac{69.5-60}{\sqrt{48}}$)	M1	$Z = \pm\frac{x - their\ 60}{\sqrt{their\ 48}}$
	= $\Phi(1.371)$	M1	69.5 or 70.5 seen in an attempted standardisation expression as cc
	=0.915	A1	Correct final answer
		4	
(iii)	np = 60, nq = 240: both > 5, (so normal approximation holds)	B1	Both parts evaluated are required
		1	

329. 9709_m17_qp_62 Q: 3

It is found that 10% of the population enjoy watching Historical Drama on television. Use an appropriate approximation to find the probability that, out of 160 people chosen randomly, more than 17 people enjoy watching Historical Drama on television. [5]

Answer:

Question	Answer	Marks	Guidance
	$np = 160\times0.1$ (16) $npq = 160\times0.1\times0.9$ (14.4)	B1	Correct unsimplified *np* and *npq*
	$P(>17) = P\left(z > \frac{17.5-16}{\sqrt{14.4}}\right) = P(z > 0.3953)$	M1	Standardising need √
		M1	16.5 or 17.5 seen in standardised eqn for continuity correction
	= 1 – 0.6536	M1	Correct area from their mean (1 – Φ), final solution
	= 0.346	A1	
	Total:	5	

330. 9709_m17_qp_62 Q: 4

The weights in kilograms of packets of cereal were noted correct to 4 significant figures. The following stem-and-leaf diagram shows the data.

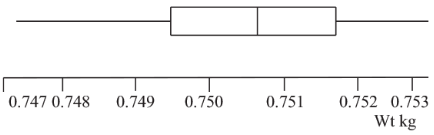
747	3	(1)
748	1 2 5 7 7 9	(6)
749	0 2 2 2 3 5 5 5 6 7 8 9	(12)
750	1 1 2 2 2 3 4 4 5 6 7 7 8 8 9	(15)
751	0 0 2 3 3 4 4 4 5 5 7 7 9	(13)
752	0 0 0 1 1 2 2 3 4 4 4	(11)
753	2	(1)

Key: 748 | 5 represents 0.7485 kg.

(i) On the grid, draw a box-and-whisker plot to represent the data. [5]

(ii) Name a distribution that might be a suitable model for the weights of this type of cereal packet. Justify your answer. [2]

Answer:

Question	Answer	Marks	Guidance
(i)	LQ = 0.7495 Med = 0.7507 UQ = 0.7517	M1	Attempt to find all 3 quartiles can be implied, Condone LQ=0.7496, Med=0.7506, UQ=0.7515
		B1	Correct median line in box using their scale
		A1	Correct quartiles in box
		B1	Correct end whiskers(not dots or boxes), lines not through box,
		B1	Correct uniform scale from at least 0.7473 to 0.7532, and label (wt) kg oe can be seen in title or scale
	Total:	5	

Question	Answer	Marks	Guidance
(ii)	Normal	B1	
	Symmetrical/peaks in middle or tails off quickly	B1	Need symm + another reason
	Total:	2	

331. 9709_m17_qp_62 Q: 7

The lengths, in centimetres, of middle fingers of women in Raneland have a normal distribution with mean μ and standard deviation σ. It is found that 25% of these women have fingers longer than 8.8 cm and 17.5% have fingers shorter than 7.7 cm.

(i) Find the values of μ and σ. [5]

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The lengths, in centimetres, of middle fingers of women in Snoland have a normal distribution with mean 7.9 and standard deviation 0.44. A random sample of 5 women from Snoland is chosen.

(ii) Find the probability that exactly 3 of these women have middle fingers shorter than 8.2 cm. [5]

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(b) The random variable X has a normal distribution with mean equal to the standard deviation. Find the probability that a particular value of X is less than 1.5 times the mean. [3]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	$0.674 = \frac{8.8-\mu}{\sigma} \Rightarrow 0.674\sigma = 8.8-\mu$	**B1**	±0.674 seen
	$-0.935 = \frac{7.7-\mu}{\sigma} \Rightarrow -0.935\sigma = 7.7-\mu$	**B1**	±0.935 seen (condone ±0.934)
		M1	An eqn with a z-value, μ and σ allow sq rt, sq cc
		M1	sensible attempt to eliminate μ or σ by substitution or subtraction
	$\sigma = 0.684$ $\mu = 8.34$	**A1**	correct answers (from –0.935)
	Total:	**5**	
(a)(ii)	$P(< 8.2) = P\left(z < \frac{8.2-7.9}{0.44}\right)$	**M1**	Standardising no cc no sq rt no sq
		M1	Correct area ie Φ, final solution
	$= P(z < 0.6818) = 0.7524$	**A1**	Correct prob rounding to 0.752
	$P(3) = {}^5C_3\,(0.7524)^3(0.2476)^2$	**M1**	Binomial ${}^5C_x$ powers summing to 5, any p, $\Sigma p = 1$
	$= 0.261$	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
(b)	$P(< 1.5\mu) = P\left(z < \frac{1.5\mu-\mu}{\mu}\right) = P\,(z < 0.5)$	***M1**	standardising with μ and σ (σ may be replaced by μ)
		DM1	just one variable
	$= 0.692$	**A1**	
	Total:	**3**	

332. 9709_s17_qp_61 Q: 6

The random variable X has a normal distribution with mean μ and standard deviation σ. You are given that $\sigma = 0.25\mu$ and $P(X < 6.8) = 0.75$.

(i) Find the value of μ. [4]

(ii) Find $P(X < 4.7)$. [3]

(b) The lengths of metal rods have a normal distribution with mean 16 cm and standard deviation 0.2 cm. Rods which are shorter than 15.75 cm or longer than 16.25 cm are not usable. Find the expected number of usable rods in a batch of 1000 rods. [4]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	$z = 0.674$	B1	rounding to ±0.674 or 0.675
	$0.674 = \frac{6.8-\mu}{0.25\mu}$	M1	standardising, no cc, no sq rt, no sq, σ may still be present on RHS
		M1	subst and sensible solving for μ must collect terms, no z-value needed can be 0.75 or 0.7734 need a value for μ
	$\mu = 5.82$	A1	
	Total:	4	
(a)(ii)	$P(X < 4.7) = P\left(z < \frac{4.7-5.819}{1.4548}\right)$	M1	± standardising no cc, no sq rt, no sq unless penalised in (a)(i)
	= ϕ(–0.769) = 1 – 0.7791	M1	correct side for their mean i.e. 1–ϕ (final solution)
	= 0.221	A1	
	Total:	3	
(b)	$P(<15.75) = P\left(z < \frac{15.75-16}{0.2}\right) = 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$ and P(>16.25) = 0.1056 by sym	*M1	Standardising for 15.75 or 16.25 no cc no sq no sq rt unless penalised in (a)(i) or (a)(ii)
	P(usable) = 1 – 0.2112 = 0.7888	B1	2ϕ– 1 OE for required prob, (final solution)
	Usable rods=1000 × 0.7888 =	DM1	Mult their prob by 1000 dep on recognisable attempt to standardise
	788 or 789	A1	
	Total:	4	

333. 9709_s17_qp_62 Q: 5

The lengths of videos of a certain popular song have a normal distribution with mean 3.9 minutes. 18% of these videos last for longer than 4.2 minutes.

(i) Find the standard deviation of the lengths of these videos. [3]

(ii) Find the probability that the length of a randomly chosen video differs from the mean by less than half a minute. [4]

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The lengths of videos of another popular song have a normal distribution with the same mean of 3.9 minutes but the standard deviation is twice the standard deviation in part **(i)**. The probability that the length of a randomly chosen video of this song differs from the mean by less than half a minute is denoted by p.

(iii) Without any further calculation, determine whether p is more than, equal to, or less than your answer to part **(ii)**. You must explain your reasoning. [2]

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Answer:

(i)	$(z =)\dfrac{4.2-3.9}{\sigma}$	M1	Standardising, not square root of σ, not σ^2
	z = 0.916 or 0.915	B1	Accept 0.915 $\leqslant \pm z \leqslant$ 0.916 seen
	σ = 0.328	A1	Correct final answer (allow 20/61 or 75/229)
	Total:	**3**	

Question	Answer	Marks	Guidance
(ii)	z = 4.4 – 3.9/their 0.328 or z = 3.4 – 3.9/their 0.328 = 1.5267 = –1.5267	M1	Standardising attempt with 3.4 or 4.4 only, allow square root of σ, or σ^2
	Φ = 0.9364	A1	0.936 $\leqslant$ Φ $\leqslant$ 0.937 or 0.063 $\leqslant$ Φ $\leqslant$ 0.064 seen
	Prob = 2Φ – 1 = 2(0.9364) – 1	M1	Correct area 2Φ – 1OE i.e. Φ = – (1 – Φ), linked to final solution
	= 0.873	A1	Correct final answer from 0.9363 $\leqslant$ Φ $\leqslant$0.9365
	Total:	**4**	
(iii)	dividing (0.5) by a larger number gives a smaller z-value or more spread out as sd larger or use of diagrams	*B1	No calculations or calculated values present e.g. (σ =)0.656 seen Reference to spread or z value required
	Prob is less than that in (ii)	DB1	Dependent upon first B1
	Total:	**2**	

334. 9709_s17_qp_63 Q: 2

The probability that George goes swimming on any day is $\frac{1}{3}$. Use an approximation to calculate the probability that in 270 days George goes swimming at least 100 times. [5]

Answer:

	$np = 270 \times 1/3 = 90$, $npq = 270 \times 1/3 \times 2/3 = 60$	**B1**	Correct unsimplified *np* and *npq*, SOI
	$P(x > 100) = P\left(z > \frac{99.5 - 90}{\sqrt{60}}\right) = P(z > 1.2264)$	**M1** **M1**	±Standardising using 100 need sq rt Continuity correction, 99.5 or 100.5 used
	$= 1 - 0.8899$	**M1**	Correct area 1 – Φ implied by final prob. < 0.5
	$= 0.110$	**A1**	
	Total:	**5**	

335. 9709_s17_qp_63 Q: 4

(a) The random variable X has the distribution $N(\mu, \sigma^2)$, where $\mu = 1.5\sigma$. A random value of X is chosen. Find the probability that this value of X is greater than 0. [3]

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(b) The life of a particular type of torch battery is normally distributed with mean 120 hours and standard deviation s hours. It is known that 87.5% of these batteries last longer than 70 hours. Find the value of s. [3]

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Answer:

(a)	$P(x > 0) = P\left(z > \pm\frac{0-\mu}{\sigma}\right)$ $= P\left(z > \frac{-\mu}{\mu/1.5}\right)$ or $P\left(z > \frac{-1.5\sigma}{\sigma}\right)$	**M1**	±Standardising, in terms of μ and/or σ with 0 - …. in numerator, no continuity correction, no √
	$= P(z > -1.5)$	**A1**	Obtaining z value of ±1.5 by eliminating μ and σ, SOI
	= 0.933	**A1**	
	Total:	**3**	
(b)	$z = -1.151$	**B1**	± z value rounding to 1.1 or 1.2
	$-1.151 = \frac{70-120}{s}$	**M1**	± Standardising (using 70) equated to a z-value, no cc, no squaring, no √
	σ = 43.4 or 43.5	**A1**	
	Totals:	**3**	

336. 9709_w17_qp_61 Q: 7

The weight, in grams, of pineapples is denoted by the random variable X which has a normal distribution with mean 500 and standard deviation 91.5. Pineapples weighing over 570 grams are classified as 'large'. Those weighing under 390 grams are classified as 'small' and the rest are classified as 'medium'.

(i) Find the proportions of large, small and medium pineapples. [5]

(ii) Find the weight exceeded by the heaviest 5% of pineapples. [3]

(iii) Find the value of k such that $P(k < X < 610) = 0.3$. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(<570) = P\left(z < \frac{570-500}{91.5}\right) = P(z < 0.7650)$ $= 0.7779$	**M1**	Standardising for either 570 or 390, no cc, no sq, no √
	$P(<390) = P\left(z < \frac{390-500}{91.5}\right) = P(z < -1.202)$	**A1**	One correct z value
	$= 1 - 0.8853 = 0.1147$	**A1**	One correct Φ, final solution
	Large:0.222 (0.2221) Small: 0.115 (0.1147)	**A1**	Correct small and large
	Medium: 0.663 (0.6632)	**A1FT**	Correct Medium rounding to 0.66 or ft 1 – (their small + their large)
		5	

Question	Answer	Marks	Guidance
(ii)	$1.645 = \left(\frac{x-500}{91.5}\right)$	**B1**	± 1.645 seen (critical value)
		M1	Standardising accept cc, sq, sq rt
	$x = 651$	**A1**	$650 \leqslant$ Ans $\leqslant 651$
		3	
(iii)	$P(x > 610) = 0.1147$ (symmetry)	**M1**	Attempt to find upper end prob $x > 610$ or $\Phi(x)$, ft their $P(<390)$ from **(i)**
	$0.3 + 0.1147 = 0.4147 \Rightarrow \Phi(x) = 0.5853$	**M1**	Adding 0.3 to *their* $P(x > 610)$ or subt 0.5 from $\Phi(x)$ or $0.8853 - 0.3$
	$z = 0.215$ or 0.216	**M1**	Finding $z = \Phi^{-1}(0.5853)$
	$0.215 = \frac{k-500}{91.5}$	**M1**	Standardising and solving, accept cc, sq, sq rt
	$k = 520$	**A1**	
		5	

337. 9709_w17_qp_62 Q: 5

Blank CDs are packed in boxes of 30. The probability that a blank CD is faulty is 0.04. A box is rejected if more than 2 of the blank CDs are faulty.

(i) Find the probability that a box is rejected. [3]

(ii) 280 boxes are chosen randomly. Use an approximation to find the probability that at least 30 of these boxes are rejected. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	*EITHER:* P(> 2) = 1 – P(0, 1, 2)	(M1	Binomial term of form $^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= 1 - (0.96)^{30} - {}^{30}C_1(0.04)(0.96)^{29} - {}^{30}C_2(0.04)^2(0.96)^{28}$ (= 1 – 0.2938… – 0.3673… – 0.2219…)	A1	Correct unsimplified answer
	= 1-0.883103 = 0.117 (0.116896)	A1)	
	OR: P(> 2) = P(3,4,5,6,….30)	(M1	Binomial term of form $^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= {}^{30}C_3(0.04)^3(0.96)^{27} + {}^{30}C_4(0.04)^4(0.96)^{26} + \ldots + (0.04)^{30}$	A1	Correct unsimplified answer
	= 0.117	A1)	
		3	

Question	Answer	Marks	Guidance
(ii)	$np = 280 \times 0.1169 = 32.73$, $npq = 280 \times 0.1169 \times 0.8831 = 28.9$	M1 FT	Correct unsimplified np and npq, FT their p from **(i)**,
	$P(\geqslant 30) = P\left(z > \frac{29.5 - 32.73}{\sqrt{28.9}}\right) = P(z > -0.6008)$	M1	Substituting *their* μ and σ ($\sqrt{npq}$ only) into the Standardisation Formula
		M1	Using continuity correction of 29.5 or 30.5
		M1	Appropriate area Φ from standardisation formula P(z >….) in final solution
	= 0.726	A1	
		5	

338. 9709_w17_qp_62 Q: 7

In Jimpuri the weights, in kilograms, of boys aged 16 years have a normal distribution with mean 61.4 and standard deviation 12.3.

(i) Find the probability that a randomly chosen boy aged 16 years in Jimpuri weighs more than 65 kilograms. [3]

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(ii) For boys aged 16 years in Jimpuri, 25% have a weight between 65 kilograms and k kilograms, where k is greater than 65. Find k. [4]

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In Brigville the weights, in kilograms, of boys aged 16 years have a normal distribution. 99% of the boys weigh less than 97.2 kilograms and 33% of the boys weigh less than 55.2 kilograms.

(iii) Find the mean and standard deviation of the weights of boys aged 16 years in Brigville. [5]

Answer:

Question	Answer	Marks	Guidance
(i)	$P(>65) = P\left(z > \frac{65-61.4}{12.3}\right) = P(z > 0.2927)$	M1	Standardising no continuity correction, no square or square root, condone ± standardisation formula
		M1	Correct area (< 0.5)
	$= 1 - 0.6153 = 0.385$	A1	
		3	

Question	Answer	Marks	Guidance
(ii)	$P(<65) = 0.6153$ so $P(<k) = 0.25 + 0.6153 = 0.8653$	B1	
	$z = 1.105$	B1	$z = \pm 1.105$ seen or rounding to 1.1
	$1.105 = \frac{k-61.4}{12.3}$	M1	standardising allow ±, cc, sq rt, sq. Need to see use of tables backwards so must be a z-value, not $1 - z$ value.
	$k = 75.0$	A1	Answers which round to 75.0. Condone 75 if supported.
		4	
(iii)	$2.326 = \frac{97.2-\mu}{\sigma}$	B1	± 2.326 seen (Use of critical value)
	$-0.44 = \frac{55.2-\mu}{\sigma}$	B1	± 0.44 seen
		M1	An equation with a z-value, μ, σ and 97.2 or 55.2, allow $\sqrt{\sigma}$ or σ^2
		M1	Algebraic elimination μ or σ from *their* two simultaneous equations
	$\mu = 61.9$ $\sigma = 15.2$	A1	both correct answers
		5	

339. 9709_w17_qp_63 Q: 7

Josie aims to catch a bus which departs at a fixed time every day. Josie arrives at the bus stop T minutes before the bus departs, where $T \sim N(5.3, 2.1^2)$.

(i) Find the probability that Josie has to wait longer than 6 minutes at the bus stop. [3]

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On 5% of days Josie has to wait longer than x minutes at the bus stop.

(ii) Find the value of x. [3]

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(iii) Find the probability that Josie waits longer than x minutes on fewer than 3 days in 10 days. [3]

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(iv) Find the probability that Josie misses the bus. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$P(t > 6) = P\left(z > \frac{6-5.3}{2.1}\right) = P(z > 0.333)$	M1	Standardising, no continuity correction, no sq, no sq rt
	$= 1 - 0.6304$	M1	Correct area $1 - \Phi\ (< 0.5)$, final solution
	$= 0.370$ or 0.369	A1	
		3	
(ii)	$z = 1.645$	B1	± 1.645
	$1.645 = \frac{x-5.3}{2.1}$	M1	Standardising, no continuity correction, allow sq, sq rt. Must be equated to a z-value
	$x = 8.75$ or 8.755 or 8.7545	A1	
		3	
(iii)	$n = 10, p = 0.05$	M1	Bin term ${}^{10}C_x\, p^x(1-p)^{10-x}$
	$P(0, 1, 2) = (0.95)^{10} + {}^{10}C_1(0.05)(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8$	M1	Correct unsimplified answer
	$= 0.988$ (0.9885 to 4 sf)	A1	
		3	
(iv)	P(misses bus) $= P(t < 0)$	*M1	Seeing t linked to zero
	$= P\left(z < \frac{0-5.3}{2.1}\right) = P(z < -2.524) = 1 - \Phi(2.524)$ $= 1 - 0.9942$	DM1	Standardising with $t = 0$, no continuity correction, no sq, no sq rt
	$= 0.0058$	A1	
		3	

340. 9709_m16_qp_62 Q: 7

The times taken by a garage to fit a tow bar onto a car have a normal distribution with mean m hours and standard deviation 0.35 hours. It is found that 95% of times taken are longer than 0.9 hours.

(i) Find the value of m. [3]

(ii) On one day 4 cars have a tow bar fitted. Find the probability that none of them takes more than 2 hours to fit. [5]

The times in hours taken by another garage to fit a tow bar onto a car have the distribution $N(\mu, \sigma^2)$ where $\mu = 3\sigma$.

(iii) Find the probability that it takes more than 0.6μ hours to fit a tow bar onto a randomly chosen car at this garage. [3]

Answer:

(i)	$z = -1.645$ $-1.645 = \dfrac{0.9 - m}{0.35}$ $m = 1.48$	**B1** **M1** **A1** 3	± 1.64 to 1.65 seen Standardising with a z-value accept $(0.35)^2$ Correct answer
(ii)	$P(< 2) = P\left(z < \dfrac{2 - 1.476}{0.35}\right)$ $= P(z < 1.50)$ $= 0.933$ Prob $= (0.9332)^4$ $= 0.758$	**M1** **M1** **A1** **M1** **A1** 5	Standardising no sq , FT *their m*, no cc Correct area i.e. F Accept correct to 2sf here Power of 4, from attempt at P(z) Correct answer
(iii)	$P(t > 0.6\mu) = P\left(z > \dfrac{0.6\mu - \mu}{\mu / 3}\right)$ $= P(z > -1.2)$ $= 0.885$	**M1** **M1** **A1** 3	Standardising attempt with 1 or 2 variables Eliminating μ or σ Correct final answer

341. 9709_s16_qp_61 Q: 1

The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

Answer:

Question	Answer	Marks	Guidance
	$z = 1.037$ $1.037 = \dfrac{1.8 - 1.62}{\sigma}$ $\sigma = 0.18/1.037 = 0.174$	**B1** **M1** **A1** [3]	Rounding to 1.04 Standardising attempt allow cc no sq rt must have a z-value i.e. not 0.8023 or 0.5596.

342. 9709_s16_qp_61 Q: 5

Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

Answer:

(i)	$P(x < 3.0) = P\left(z < \frac{3.0-2.6}{0.25}\right)$ + $P(z < 1.6) = 0.945$	**M1** **M1** **A1** [3]	Standardising no sq rt no cc Correct area i.e. prob > 0.5 legit
(ii)	$X \sim B(500, 0.9452) \sim N(472.6, 25.898)$ $P\left(z > \frac{479.5-472.6}{\sqrt{25.89848}}\right) = P(z > 1.3558)$ $= 1 - 0.9125 = 0.0875$	**M1** **M1** **M1** **M1** **A1** [5]	500 ×'0.9452' and 500×'0.9452'×('1 – 0.9452') seen oe Standardising must have sq rt. All M marks indep cc either 479.5 or 480.5 seen correct area i.e. < 0.5

Question	Answer	Marks	Guidance
(iii)	500× 0.9452 and 500× (1–0.9452) are both > 5	**B1**√ [1]	must see at least 500× 0.0548 > 5oe ft their (i) accept $np > 5$, $nq > 5$ if both not $npq > 5$

343. 9709_s16_qp_62 Q: 2

When visiting the dentist the probability of waiting less than 5 minutes is 0.16, and the probability of waiting less than 10 minutes is 0.88.

(i) Find the probability of waiting between 5 and 10 minutes. [1]

A random sample of 180 people who visit the dentist is chosen.

(ii) Use a suitable approximation to find the probability that more than 115 of these people wait between 5 and 10 minutes. [5]

Answer:

(i)	0.72	**B1** [1]	
(ii)	$np = 180 \times 0.72$, $npq = 180 \times 0.72 \times 0.28$ $X \sim N(129.6, 36.288)$	**B1**√	180 × 0.72, 180 × 0.72 × 0.28 seen, their values or correct
	$P(x > 115) = P\left(z > \frac{115.5-129.6}{\sqrt{36.288}}\right)$	**M1**	Standardising (±) must have sq rt
		M1	cc either 115.5 or 114.5 seen
	$= P(z > -2.341)$	**M1**	Correct area, Φ from final answer attempt fully correct method
	$= 0.990$	**A1** [5]	

344. 9709_s16_qp_62 Q: 6

The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t. [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

Answer:

(i)	$P(x > 10.2) = P\left(z > \frac{10.2-9.5}{1.3}\right)$	**M1**	Standardising allow cc, sq rt, sq
	$= P(z > 0.53846)$ $= 1 - 0.7046$ $= 0.295$	**M1** **A1** [3]	1 – Φ final solution attempt
(ii)	$z = -1.282$	**B1**	± rounding to 1.28 seen
	$-1.282 = \frac{t-9.5}{1.3}$	**M1**	Standardising correctly can be ± z value here
	$t = 7.83$	**A1** [3]	Correct answer from $z = -1.282$ only
(iii)	$P(x < 8.8) = 0.2954$ by symmetry Days $= 365 \times 0.2954$ $= 107$ or 108	**B1** **M1** **A1** [3]	oe method, FT *their 0.2954 from (i)* Mult a probability <1 by 365 Correct answer (no decimals)

345. 9709_s16_qp_63 Q: 5

The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

(i) Find the value of σ. [3]

When Jodu sits at a desk, his knees are at a height of 58 cm above the floor. A desk is comfortable for Jodu if his knees are at least 9 cm below the top of the desk. Jodu's school has 300 desks.

(ii) Calculate an estimate of the number of these desks that are comfortable for Jodu. [5]

Answer:

(i)	$z = 1.015$ $1.015 = \frac{70-69}{\sigma}$ $\sigma = 0.985$ (200/203)	B1 M1 A1 [3]	Accept z between ±1.01 and 1.02 Standardising
(ii)	$58 + 9 = 67$ $P(> 67) = P\left(z > \frac{67-69}{0.9852}\right)$ $= P(z > -2.03)$ $= 0.9788$ 300×0.9788 $= 293.6$ so 293	M1 M1 M1 M1 A1 [5]	58 + 9 seen or implied (or 69-58 or 69-9) Standardising ± z no cc allow their sd (must be +ve) Alt. 1 69-58 =11, $P(>9)=P\left(z > \frac{9-11}{0.9852}\right)$ Alt.2 69-9 =60, $P(>58) =P\left(z > \frac{58-60}{0.9852}\right)$ Correct prob area Multiply their prob (from use of tables) by 300 – accept 293 or 294 from fully correct working

346. 9709_s16_qp_63 Q: 7

Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

Answer:

(i)	$^{12}C_8\,(0.65)^8(0.35)^4 + {}^{12}C_9\,(0.65)^9(0.35)^3 + {}^{12}C_{10}\,(0.65)^{10}(0.35)^2$	M1	Bin term with $^{12}C_r p^r(1-p)^{12-r}$ seen $r \neq 0$ any $p<1$
		M1	Summing 2 or 3 bin probs p = 0.65 or 0.35, n = 12
	= 0.541	A1 [3]	
(ii)	$P\left(\bar{R}\bar{R}\bar{R}R\right) = 0.35 \times 0.35 \times 0.35 \times 0.65$	M1	Mult 4 probs either $(0.35)^3(0.65)$ or $(0.65)^3(0.35)$
	= 0.0279	A1 [2]	
(iii)	P(7) = 0.2039 (unsimplified)	B1	$^{12}C_7\,(0.65)^7(0.35)^5$
	Mean = 250×'0.2039' (= 50.9798) Var = 250×'0.2039' × '(1 – 0.2039)' (= 40.5851)	B1	Correct unsimplified np and npq using 'their 0.2039' but not 0.65 or 0.35
	$P(>54) = P\left(\frac{54.5-50.9798}{\sqrt{40.5851}}\right)$	M1	Standardising need sq rt – must be from working with 54
	= P(z > 0.5526)	M1	cc either 53.5 or 54.5
	= 1 – Φ(0.5526) = 1 – 0.7098	M1	correct area < 0.5 i.e. 1 – Φ - must be from working with 54
	= 0.290	A1 [6]	

347. 9709_w16_qp_61 Q: 4

Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

Answer:

(i)	$P(<1) = P\left(z < \frac{1-1.04}{0.017}\right) = P(z < -2.353)$ $= 1 - 0.9907$ $= 0.0093$	**M1** **M1** **A1**	[3]	Standardising no cc, no √ or sq 1 – Φ (final process)
(ii)	expected number 1000 ÷ 1.04 = 961 or 962	**B1**	[1]	Or anything in between
(iii)	$z = -1.765$ $-1.765 = \frac{1-\mu}{0.017}$ $=1.03$	**B1** **M1** **A1**	[3]	± 1.76 to 1.77 Standardising must have a z-value, allow √ or sq
(iv)	expected number = 1000 ÷ 1.03 = 971 or 970	**B1**√	[1]	Or anything in between, ft their (iii)

348. 9709_w16_qp_62 Q: 3

On any day at noon, the probabilities that Kersley is asleep or studying are 0.2 and 0.6 respectively.

(i) Find the probability that, in any 7-day period, Kersley is either asleep or studying at noon on at least 6 days. [3]

(ii) Use an approximation to find the probability that, in any period of 100 days, Kersley is asleep at noon on at most 30 days. [5]

Answer:

(i)	Bin (7, 0.8) $P(6, 7) = {}^7C_6\,(0.8)^6(0.2)^1 + (0.8)^7$ $= 0.577$	**M1** **M1** **A1**	[3]	${}^7C_n\,p^n(1-p)^{7-n}$ seen Correct unsimplified expression for P(6,7)
(ii)	mean = 100×0.2 = 20 Var = 100×0.2×0.8 = 16 $P(\text{at most } 30) = P\left(z < \frac{30.5-20}{\sqrt{16}}\right)$ $= P(z < 2.625)$ $= 0.996$	**B1** **M1** **M1** **M1** **A1**	[5]	Correct unsimplified mean and var Standardising must have sq rt, their μ, variance cc either 29.5 or 30.5 Correct area Φ , from final process

349. 9709_w16_qp_62 Q: 4

The time taken to cook an egg by people living in a certain town has a normal distribution with mean 4.2 minutes and standard deviation 0.6 minutes.

(i) Find the probability that a person chosen at random takes between 3.5 and 4.5 minutes to cook an egg. [3]

12% of people take more than t minutes to cook an egg.

(ii) Find the value of t. [3]

(iii) A random sample of n people is taken. Find the smallest possible value of n if the probability that none of these people takes more than t minutes to cook an egg is less than 0.003. [3]

Answer:

(i)	$P(<4.5) = P\left(z < \frac{4.5-4.2}{0.6}\right) = P(z<0.5)$ $= 0.6915$ $P(<3.5) = P\left(z < \frac{3.5-4.2}{0.6}\right) = P(z< -1.167)$ $= 1 - 0.8784 = 0.1216$ $0.6915 - 0.1216 = 0.570$	**M1** **M1** **A1**	 [3]	Standardising once no cc no sq no sq rt $\Phi_1 - (1 - \Phi_2)$ $[P_1 - P_2,\ 1>P_1>0.5,\ 0.5>P_2>0]$ oe
(ii)	$z = 1.175$ $1.175 = \frac{t-4.2}{0.6}$ $t = 4.91$	**B1** **M1** **A1**	 [3]	±1.17 to 1.18 seen Standardising no cc, allow sq, sq rt with z – value (not ±0.8106, 0.5478, 0.4522, 0.1894, 0.175 etc.) Correct answer from $z = 1.175$ seen (4sf)
(iii)	$(0.88)^n < 0.003$ $n > \lg(0.003)/\lg(0.88)$ $n > 45.4$ $n = 46$	**M1** **M1** **A1**	 [3]	Inequality or eqn in 0.88, power correctly placed using n or $(n\pm1)$, 0.003 or $(1 - 0.003)$ oe Attempt to solve by logs or trial and error (may be implied by answer) Correct integer answer

350. 9709_w16_qp_63 Q: 6

The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) **(a)** Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

Answer:

(i)	P(small) = $P\left(z < \frac{95-150}{50}\right)$ = P(z < –1.1) = 1 – 0.8643 = 0.136	M1 M1 A1	 [3]	± standardising using 95, no cc, no sq, no sq rt 1 – Φ (in final answer)
(ii)	z = 1.282 $1.282 = \frac{x-150}{50}$ x = 214 g	B1 M1 A1	 [3]	± rounding to 1.28 Standardised eqn in their z allow cc
(iii)	P(small) = 0.1357, P(large) = 0.1357 symmetry P(medium) = 1 – 0.1357×2 = 0.7286 **AG**	B1	[1]	Correct answer legit obtained
(b)	Expected cost per banana = 0.1357×10 + 0.1357×25 + 0.7286×20 = 19.3215 cents Total cost of 100 bananas = 1930 (cents) ($19.30)	*M1 DM1 A1	 [3]	Attempt at multiplying each 'prob' by a price and summing Mult by 100

351. 9709_w16_qp_63 Q: 7

Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

(i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]

(ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

(iii) Find the probability that Annabel eats potato on more than 44 days in a year of 365 days. [5]

Answer:

(i)	$P(2) = {}^7C_2(0.1)^2(0.9)^5$ = 0.124	M1 A1	 [2]	Bin term ${}^7C_2p^2(1-p)^5 \quad 0 < p < 1$
(ii)	$(0.15)^1(0.1)^2(0.75)^2 \times 5!/2!2!$ = 0.0253 or 81/3200	M1 M1 A1	 [3]	Mult probs for options, $(0.15)^a(0.1)^b(0.75)^c$ where a + b + c sum to 5 Mult by 5!/2!2! oe
(iii)	mean = 365×0.15 (= 54.75 or 219/4) Var = 365× 0.15×0.85 (= 46.5375 or 3723/80) $P(x > 44) = P\left(z > \frac{44.5-54.75}{\sqrt{46.5375}}\right)$ = P(z > –1.5025) = 0.933	B1 M1 M1 M1 A1	 [5]	Correct unsimplified mean **and** var, oe ± Standardising need sq rt cc either 44.5 (or 43.5) Φ Correct answer accept 0.934

352. 9709_s15_qp_61 Q: 1

The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ. [4]

Answer:

	$P(x < 3.273) = 0.5 - 0.475 = 0.025$	**M1**	Attempt to find z-value using tables in reverse
	$z = -1.96$	**A1**	± 1.96 seen
	$\dfrac{3.2-\mu}{0.714} = -1.96$	**M1**	Solving their standardised equation z-value not nec
	$\mu = 4.60$s	**A1 [4]**	Correct ans accept 4.6

353. 9709_s15_qp_61 Q: 6

(i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

Answer:

(i)	$P(5, 6, 7) = {}^8C_5(0.68)^5(0.32)^3 + {}^8C_6(0.68)^6(0.32)^2 + {}^8C_7(0.68)^7(0.32)$	M1 M1 A1	Binomial term ${}^8C_x\, p^x(1-p)^{8-x}$ seen $0 < p < 1$ Summing 3 binomial terms Correct unsimplified answer
	$= 0.722$	A1 **[4]**	Correct answer
(ii)	$np = 340$, $npq = 108.8$	B1	Correct (unsimplified) mean and var
	$P(x > 337) = P\left(z > \dfrac{337.5-340}{\sqrt{108.8}}\right)$	M1 M1	standardising with sq rt must have used 500 cc either 337.5 or 336.5
	$= P(z > -0.2396)$ $= 0.595$	M1 A1 **[5]**	correct area (> 0.5) must have used 500 correct answer
(iii)	$np\,(340) > 5$ and $nq(160) > 5$	B1 **[1]**	must have both or at least the smaller, need numerical justification

354. 9709_s15_qp_62 Q: 7

(a) Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

(i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

(ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

Answer:

	total = 1210	A1	Correct final answer
(a) (i)	prob = $p\left(z < \frac{30-35.2}{4.7}\right)$ $= P(z < -1.106)$ $= 1 - 0.8655 = 0.1345$ $0.1345 \times 52 = 6.99$	M1 M1 A1 A1 **4**	Standardising no sq rt no cc no sq $1 - \Phi$ Correct ans rounding to 0.13 Correct final answer accept 6 or 7 if 6.99 not seen but previous prob 0,1345 correct
(ii)	$\Phi(t) = 0.648$ $z = 0.380$ $0.380 = \frac{t - 35.2}{4.7}$ $t = 37.0$	B1 M1 A1 **3**	0.648 seen standardising allow cc, sq rt,sq, need use of tables not 0.148, 0.648, 0.352, 0.852 correct answer rounding to 37.0
(b)	$\frac{7-\mu = -0.8\sigma}{\sigma}$ so $7 - \mu = -0.8\sigma$ $\frac{10-\mu}{\sigma} = 0.44$ so $10 - \mu = 0.44\sigma$ $\mu = 8.94$ $\sigma = 2.42$	B1 B1 M1 M1 A1 **5**	± 0.8 seen ± 0.44 seen An eqn with z-value, μ and σ no sq rt no cc no sq Sensible attempt to eliminate μ or σ by subst or subtraction, need at least one value Correct answers

355. 9709_s15_qp_63 Q: 1

The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

Answer:

	z = 1.136 $1.136 = \frac{195 - \mu}{22}$ $\mu = 170$	B1 M1 A1 **[3]**	± 1.136 seen, not ±1.14, Standardising, no cc no sq rt, equated to their z not 0.128 or 0.872 Correct answer, nfww

356. 9709_s15_qp_63 Q: 3

On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

Answer:

	$\mu = 300 \times 0.072 = 21.6$, $\sigma^2 = 20.0448$ $P(x < 18) = P\left(z < \frac{17.5 - 21.6}{\sqrt{20.0448}}\right)$ $= P(z < -0.9157)$ $= 1 - 0.8201$ $= 0.180$	B1 M1 M1 M1 A1 **[5]**	300×0.072 seen and 300×0.072×0.928 seen or implied ($\sigma = 4.4771, \sigma^2 = 20(.0)$) oe ±Standardising, their mean/var, with sq root Cont corr 17.5 or 18.5 Correct area 1 - Φ Answer wrt 0.180, nfww

357. 9709_s15_qp_63 Q: 5

The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as 'large'.

(i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

(ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

Answer:

(i)	P(large) = $1 - \Phi\left(\frac{29-21.7}{6.5}\right)$ $= 1 - \Phi(1.123) = 1 - 0.8692$ $= 0.1308$ $P(0,1) = (0.8692)^8 + {}^8C_1(0.1308)(0.8692)^7$ $= 0.718$	M1 M1 A1 M1 M1 A1 **[6]**	Standardising no cc no sq rt Correct area 1 – Φ Rounding to 0.13 Any bin term with ${}^8C_x p^x(1-p)^{8-x}$ $0 < p < 1$ Summing bin P(0) + P(1) only with $n = 8$, oe Correct ans
(ii)	$= 1 - (0.8692)^n > 0.98$ $(0.8692)^n < 0.02$ Least number = 28	M1 M1 A1 **[3]**	eq/ineq involving their $(0.8692)^n$ or $(0.1308)^n$, 0.02 or 0.98 oe with or without a 1 solving attempt (could be trial and error) – may be implied by their answer correct answer

358. 9709_w15_qp_61 Q: 2

The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

Answer:

	$\mu = 54.1$ $z = -1.11$ $-1.11 = \frac{50.9 - 54.1}{\sigma}$ $\sigma = 2.88$	**B1** **B1** **M1** **A1** [4]	Stated or evaluated Accept rounding to ± 1.1 Standardising no cc no sq rt Correct answer

359. 9709_w15_qp_61 Q: 7

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

Answer:

(i)	let P(2, 4, 6) all = p then P(1, 3, 5) all = 2p $3p + 6p = 1$ $p = 1/9$ so prob (3) = 2/9 (0.222)	**M1** **M1** **A1** [3]	Using P(even) = 2P(odd) or vice versa oe Summing P(odd+ even) or P(1, 2, 3, 4, 5, 6) = 1 Correct answer
(ii)	$P(5, 5, 6) = 2/9 \times 2/9 \times 1/9 \times {}^3C_2$ = 4/243 (0.0165)	**M1** **M1** **A1** [3]	Mult three probs together Mult by 3 oe ie summing 3 options Correct answer
(iii)	$\mu = 100\times1/3 = 33.3$, $\sigma = 100\times1/3\times2/3 = 22.2$ $P(x \leqslant 37) = P\left(z \leqslant \dfrac{37.5 - \frac{100}{3}}{\sqrt{\frac{200}{9}}}\right) = P(z \leqslant 0.8839)$ = 0.812	**B1** **M1** **M1** **M1** **A1** [5]	Unsimplified 100/3 and 200/9 seen Standardising need sq rt 36.5 or 37.5 seen correct area using their mean Correct answer

360. 9709_w15_qp_62 Q: 7

(a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

Answer:

(a) (i)	$P(x > 3900) = P\left(z > \frac{3900 - 4520}{560}\right)$ $= P(z > -1.107) = \Phi(1.107)$ $= 0.8657$ Number of days = $365 \times 0.0.8657$ $= 315$ or 316 (315.98)	**M1** **M1** **A1** **B1**√ 4	Standardising no cc no sq rt no sq Correct area Φ ie > 0.5 Prob rounding to 0.866 Correct answer ft their wrong prob if previous A0, $p < 1$, ft must be accurate to 3sf
(ii)	$z = 1.165$ $1.165 = \frac{8000 - m}{560}$ $m = 7350$ (7347.6)	**B1** **M1** **A1** 3	± 1.165 seen Standardising eqn allow sq, sq rt, cc, must have z-value eg not 0.122, 0.878, 0.549, 0.810. Correct answer rounding to 7350
(iii)	$P(0, 1) = (0.878)^6 + {}^6C_1(0.122)^1(0.878)^5$ $= 0.840$ accept 0.84 Normal approx. to Binomial. M0, M0, A0	**M1** **M1** **A1** 3	Binomial term ${}^6C_x\, p^x(1-p)^{6-x}$ $0 < p < 1$ seen Correct unsimplified expression Correct answer
(b)	$P(< 2\mu) = P\left(z > \frac{2\mu - \mu}{\sigma}\right) = P(z < 1.5)$ $= 0.933$	**M1** **M1** **A1** 3	Standardising with μ and σ Attempt at one variable and cancel Correct answer

361. 9709_w15_qp_63 Q: 4

The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

(ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

Answer:

(i)	$z = 1.127$ $1.127 = \frac{136 - 125}{\sigma}$ $\sigma = 9.76$	**B1** **M1** **A1** 3	± 1.127 seen accept rounding to ±1.13 Standardising no cc no sq rt, with attempt at z (not ±0.8078, ±0.5517, ±0.13, ±0.87) Correct ans
(ii)	$P(131 < x < 141) = P\left(\frac{131-125}{9.76} < z < \frac{141-125}{9.76}\right)$ $= \Phi(1.639) - \Phi(0.6147)$ $= 0.9493 - 0.7307$ $= 0.2186$ Number = $0.2186 \times 170 = 37$ or 38 or awrt 37.2	**M1** **M1** **M1** **A1** 4	Standardising once with their sd, no √, ², allow cc Correct area Φ2 – Φ1 Mult by 170, P<1 Correct answer, nfww

362. 9709_w15_qp_63 Q: 7

A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]

Answer:

(i)	P(0, 1, 2) = $(0.92)^{19}+{}^{19}C_1(0.08)(0.92)^{18}+{}^{19}C_2(0.08)^2(0.92)^{17}$	**M1** **M1**		Binomial term ${}^{19}C_x p^x(1-p)^{19-x}$ seen $0<p<1$ Correct unsimplified expression
	= 0.809	**A1**	3	Correct answer (no working SC B2)
(ii)	P(at least 1) = 1 – P(0) $= 1 - P(0.92)^n > 0.90$ $0.1 > (0.92)^n$ $n > 27.6$	**M1** **M1**		Eqn with their 0.92^n, 0.9 or 0.1, 1 not nec Solving attempt by logs or trial and error, power eqn with one unknown power
	Ans 28	**A1**	3	Correct answer, not approx., ≈, ⩾, >, ⩽, <
(iii)	np = 1800 × 0.08 = 144 npq = 132.48	**B1**		correct unsimplified np and npq seen accept 132.5, 132, 11.5, awrt 11.51
	P(at least 152) = $P\left(z > \left(\frac{151.5-144}{\sqrt{132.48}}\right)\right)$	**M1** **M1**		standardising, with √ cont correction 151.5 or 152.5 seen
	$= P(z > 0.6516)$ $= 1 - 0.7429$ $= 0.257$	**M1** **A1**	5	correct area 1 – Φ (probability) correct answer
(iv)	Use because 1800 ×0.08 (and 1800 × 0.92 are both) > 5	**B1**	1	1800 ×0.08 > 5 is sufficient $np>5$ is sufficient if clearly evaluated in (iii) If $npq>5$ stated then award B0