Sequences and Series

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Sequences and Series
Booklet	Question Paper

Time Allowed: **125 minutes**

Score: **/104**

Percentage: **/100**

Grade Boundaries:

1 The coefficient of x^3 in the expansion of $(2+ax)^5$ is 10 times the coefficient of x^2 in $\left(1+\frac{ax}{3}\right)^4$. Find a.

[4]

2 An arithmetic progression has first term a and common difference d. The first, ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio r, where $r \neq 1$.

(i) Find d in terms of a and show that $r = \frac{5}{8}$. [7

(ii) Find the sum to infinity of the geometric progression in terms of a. [2

3 It is given that x, 6 and $x + 5$ are consecutive terms of a geometric progression.

(i) Show that $x^2 + 5x - 36 = 0$ and find the possible values of x. [3]

(ii) Hence find the possible values of the common ratio. [2]

Furthermore, x, 6 and $x + 5$ are the second, third and fourth terms of a geometric progression for which the sum to infinity exists.

(iii) Find the first term and the sum to infinity. [4]

4 An arithmetic progression has first term 5 and common difference 7.

(i) Find the value of the 10th term. [1]

(ii) Find the sum of the first 15 terms. [2]

The terms of the progression are given by $x_1, x_2, x_3, \ldots$.

(iii) Evaluate $\sum_{n=1}^{15}(2x_n + 1)$. [3]

5 Find the coefficient of x^3 in the expansion of $(1 - 2x)^5$. [4]

6 **(i)** An arithmetic sequence has first term 3 and common difference 2. Find the twenty-first term of this sequence. [2]

(ii) Find the sum to infinity of a geometric progression with first term 162 and second term 54. [3]

(iii) A sequence is given by the recurrence relation $u_1 = 3$, $u_{n+1} = 2 - u_n$, $n = 1, 2, 3, \ldots$. Find u_2, u_3, u_4, u_5 and describe the behaviour of this sequence. [2]

7 Evaluate the following, giving your answers in exact form.

(i) $\sum_{n=1}^{30} \frac{1}{n} - \sum_{n=2}^{29} \frac{1}{n}$. [2]

(ii) $\sum_{n=1}^{100} n \times (-1)^n$. [2]

8 The first term of a geometric progression is 16 and the common ratio is 0.8.

(i) Calculate the sum of the first 12 terms. [3]

(ii) Find the sum to infinity. [2]

9 **(i)** Show that the first three terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$ are $1 - x - \frac{1}{2}x^2$ and find the next term. [4]

(ii) State the range of values of x for which this expansion is valid. [1]

(iii) Hence show that the first four terms in the expansion of $(2 + x)(1 - 2x)^{\frac{1}{2}}$ are $2 - x + ax^2 + bx^3$ and state the values of a and b. [4]

10 **(i)** An arithmetic sequence has first term 5 and fifth term 37.

(a) Find an expression for u_n, the nth term of the sequence, in terms of n. [4]

(b) Find an expression for S_n, the sum of the first n terms of this sequence, in terms of n. [2]

(ii) Hence, or otherwise, calculate $\sum_{n=5}^{25}(8n-3)$. [2]

11 **(i)** Find and simplify the first three terms in the expansion of $(1-4a)^{\frac{1}{2}}$ in ascending powers of a, where $|a| < \frac{1}{4}$. [4]

(ii) Hence show that the roots of the quadratic equation $x^2 - x + a = 0$ are approximately $1 - a - a^2$ and $a + a^2$, where a is small. [4]

12 An arithmetic progression has first term a and common difference d. The first, ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio r, where $r \neq 1$.

(i) Find d in terms of a and show that $r = \frac{5}{8}$. [7]

(ii) Find the sum to infinity of the geometric progression in terms of a. [2]

13 A geometric progression with common ratio r consists of positive terms. The sum of the first four terms is five times the sum of the first two terms.

(i) Find an equation in r and deduce that $r = 2$. [3]

(ii) Given that the fifth term is 192, find the value of the first term. [1]

(iii) Find the smallest value of n such that the sum of the first n terms of the progression exceeds 10^{64}. [3]

14 Let $f(x) = \dfrac{1 + x^2}{\sqrt{4 - 3x}}$.

(i) Obtain in ascending powers of x the first three terms in the expansion of $\dfrac{1}{\sqrt{4 - 3x}}$ and state the values of x for which this expansion is valid. [5]

(ii) Hence obtain an approximation to $f(x)$ in the form $a + bx + cx^2$ where a, b and c are constants. [2]

(iii) Use your approximation to estimate $\int_0^{0.1} f(x)\,dx$. [2]

15 An arithmetic progression has 13th term equal to 60 and 31st term equal to 141.

(i) Find the first term and common difference of the progression. [3]

A second arithmetic progression has first term 1.5 and common difference 3.

(ii) **(a)** Write down the first four terms of each progression. [1]

(b) Prove that the two progressions have an infinite number of terms in common. [2]