Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*3675531203*

**FURTHER MATHEMATICS** **9231/23**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

**1** **(a)** By differentiating $e^{-x^2}$, find the Maclaurin's series for $e^{-x^2}$ up to and including the term in $x^2$. [5]

**(b)** Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2}\,dx$, giving your answer as a rational fraction in its lowest terms. [2]

**2** The variables $x$ and $y$ are related by the differential equation

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

**(a)** Find the general solution for $y$ in terms of $x$. [6]

**(b)** State an approximate solution for large positive values of $x$. [1]

3 **(a)** Show that the system of equations

$$\begin{aligned} x-2y-4z &= 1, \\ x-2y+kz &= 1, \\ -x+2y+2z &= 1, \end{aligned}$$

where $k$ is a constant, does not have a unique solution. [2]

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**(b)** Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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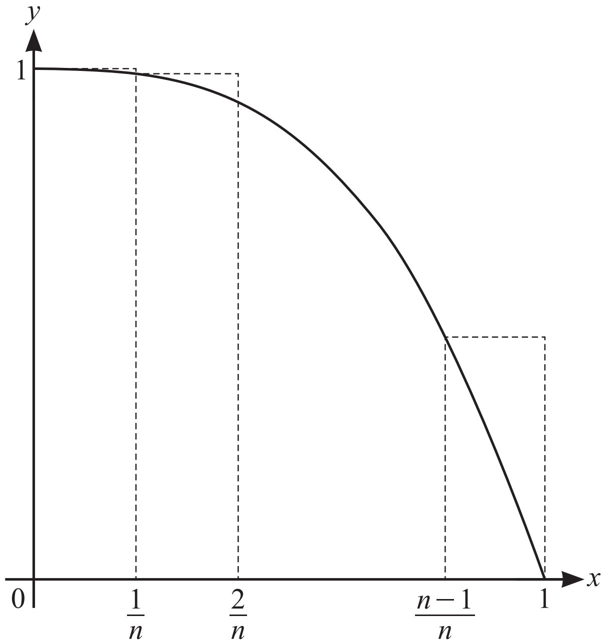
**(c)** Given instead that $k = -2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

**(d)** For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

**4**




The diagram shows the curve with equation $y = 1 - x^3$ for $0 \leqslant x \leqslant 1$, together with a set of $n$ rectangles of width $\frac{1}{n}$.

**(a)** By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3)\,dx \leqslant \frac{3n^2 + 2n - 1}{4n^2}. \qquad [4]$$

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(b) Use a similar method to find, in terms of $n$, a lower bound for $\int_0^1 (1-x^3)\,dx$. [4]

**5** It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

**(a)** By differentiating $\cos y$ with respect to $t$, show that $\dfrac{dy}{dt} = -\dfrac{1}{\sqrt{1-t^2}}$. [4]

**(b)** Find $\dfrac{d^2y}{dx^2}$ in terms of $t$, simplifying your answer. [5]

**6** **(a)** Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [5]

**(b)** Find the solution of the differential equation

$$\frac{dy}{d\theta} + y\cot\theta = \sin^3\theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$. [6]

**7** The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

**(a)** State the eigenvalues of $\mathbf{P}$. [1]

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**(b)** Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

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The $3 \times 3$ matrix $\mathbf{A}$ has distinct eigenvalues $b$, $-1$, $1$ with corresponding eigenvectors

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \begin{pmatrix}4\\-1\\0\end{pmatrix}, \quad \begin{pmatrix}2\\1\\2\end{pmatrix},$$

respectively.

**(c)** Find $\mathbf{A}$ in terms of $b$. [4]

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**8** **(a)** Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

**(b)** Starting from the definitions of coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1.$$ [3]

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The curve $C$ has equation $y = \ln\coth\left(\frac{1}{2}x\right)$ for $x > 0$.

**(c)** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosech} x$. [3]

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**(d)** It is given that the arc length of $C$ from $x = a$ to $x = 2a$ is $\ln 4$, where $a$ is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of $a$. [7]

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## Additional Page

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