Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2330973605

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

3 **(a)** Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

4 Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

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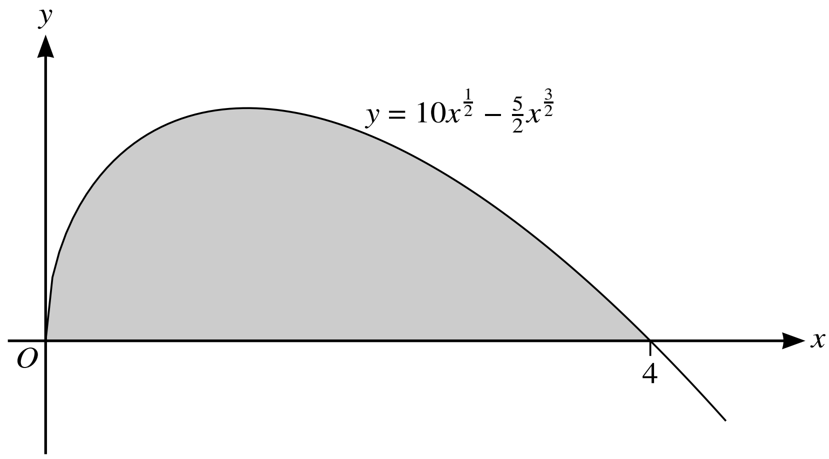

The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points (0, 0) and (4, 0).

Find the area of the shaded region. [4]

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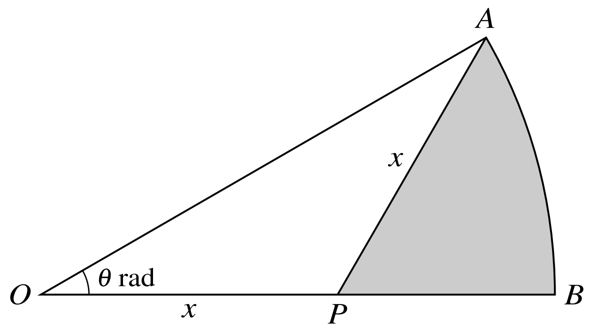

The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

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(b) Find the area of the shaded region APB in terms of x and θ. [4]

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7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. [3]

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(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta + \sin\theta} + \frac{1 - \cos\theta}{\cos\theta - \sin\theta} = 2(\cos\theta + \sin\theta - 1)$$

for $0 \leqslant \theta \leqslant \pi$. [3]

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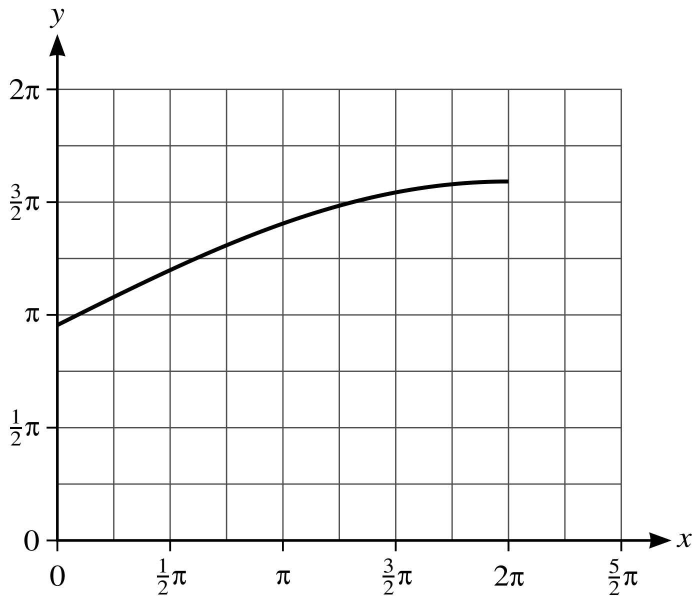

The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

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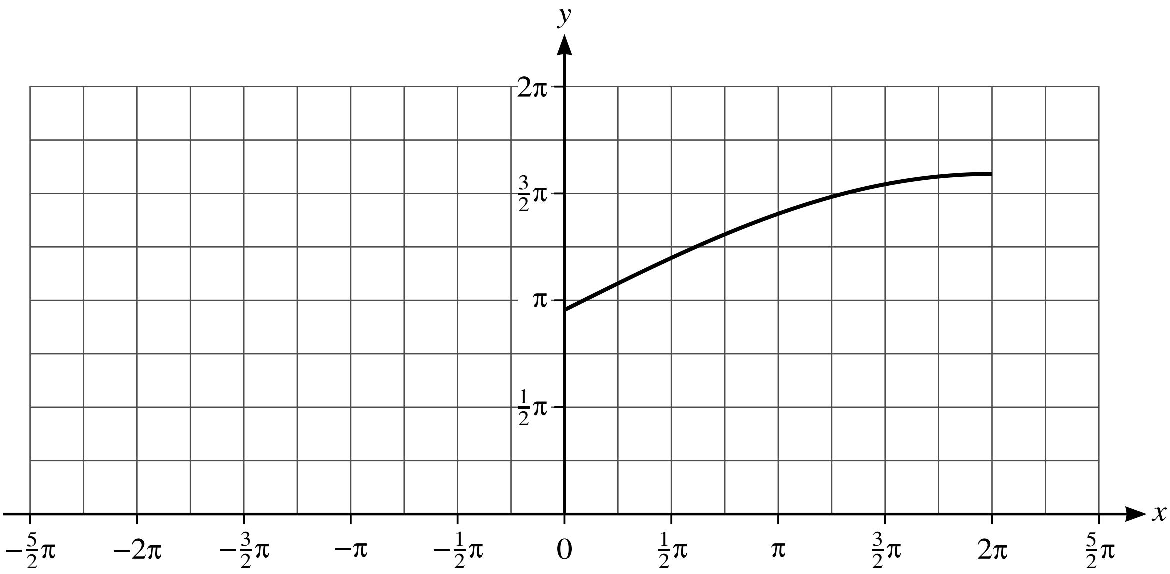
(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

10 The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

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(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

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11 The equation of a curve is

$$y = k\sqrt{4x + 1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

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(b) Find the x-coordinate of the stationary point in terms of k. [2]

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(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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