Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

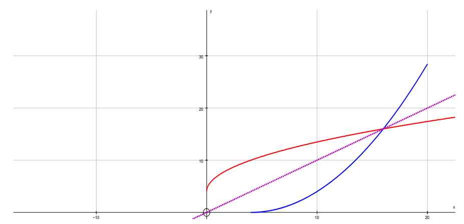
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

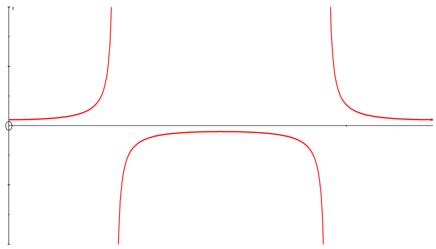
GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Guidance
1	$5x+3<-(3x-1)$ (if $(3x-1)<0$)	**M1**	Alternative $(5x+3)^2<$ or $=(3x-1)^2$ M1 $x=-2$ or $x=-\frac{1}{4}$ A1 $x<-\frac{1}{4}$ only M1A1
	$x<-\frac{1}{4}$	**A1**	Obtain at least the critical value
	$5x+3<3x-1$ (if $3x-1>0$) $\Rightarrow x<-2$	**B1**	
	$x<-\frac{1}{4}$ only	**A1**	
2(i)	$\mathrm{f}(x)\geqslant 4$	**B1**	Accept $y\geqslant 4$ but not x.
2(ii)	16	**B1**	
2(iii)	$\left(\frac{y-4}{3}\right)^2=x$	**M1**	
	$\mathrm{f}^{-1}(x)=\frac{(x-4)^2}{9}$	**A1**	Accept $y=\mathrm{f}(x)$.
2(iv)		**B2**	**B1** for general shape of $y=\mathrm{f}(x)$ starting at (0, 4) **B1** for general shape of inverse starting at approximately (4, 0) and a suggestion of intersection.
	State Reflection in line $y=x$	**B1**	
3	$z=1$ is a root implies $(z-1)$ is a factor	**B1**	Must be stated or used.
	$\frac{z^3-1}{z-1}=z^2+z+1$	**M1**	Must reach a 3 term quadratic. Accept alternative methods e.g. coefficient matching
	$z=\frac{-1\pm\sqrt{-3}}{2}$	**M1**	Allow one error of substitution in correct quadratic formula if stated. If formula not stated and error present M0
	$z=\frac{-1\pm \mathrm{i}\sqrt{3}}{2}$	**A1**	

Question	Answer	Marks	Guidance
4(i)		B1	Award for 3 branches of roughly correct shape with two asymptotes
4(ii)	$\frac{1}{\cos\theta} = \frac{1}{\sin\theta}$	M1	Allow equivalent methods
	$\tan\theta = 1$	M1	
	$\frac{\pi}{4}$ or 45°	A1	
	$\frac{5}{4}\pi$	A1	A0 if more angles given or if in degrees
5(i)	$\frac{dy}{dx} = 3x^2 + 4x - 13$	M1	Differentiate by showing a decrease in power by 1 in at least two terms.
	At $x = 2$ $m = 7$	M1	Substitute $x = 2$ in their derivative and a numerical result.
	$y = 7x - 14$	A1	
	$y = 7(-6) - 14 = -56$ $y = (-6)^3 + 2(-6)^2 - 13(-6) + 10$ $= -56$ OR use $x^3 + 2x^2 - 20x + 24$ $= -216 + 72 + 120 + 24 = 0$	A1	Confirm equality

Question	Answer	Marks	Guidance
5(ii)	$x^3+2x^2-13x+10-(7x-14)$	M1	Intention shown to subtract curve – tangent even as two separate integrals Main Alternative Show a complete method: (area of triangle from –6 to 2) – (integral from –5 to –6) + (integral from –5 to 1) – (area of integral from 1 to 2)
	$\int_a^b (x^3+2x^2-20x+24)\mathrm{d}x$ OR $\int_a^b (x^3+2x^2-13x+10)\mathrm{d}x$ OR $\int_a^b (x^3+2x^2-13x+66)\mathrm{d}x$	M1	Integrate one or other cubic expressions
	$\left[\frac{1}{4}x^4+\frac{2}{3}x^3-10x^2+24x\right]_a^b$ OR $\left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{13}{2}x^2+10x\right]_a^b$	A1	
	F(b) – F(a)	M1	Evaluate their integral with their a and b as F(b) – F(a) only. Convincing substitution of limits or evaluation of areas required.
	Show use of correct limits $\left[\frac{1}{4}x^4+\frac{2}{3}x^3-10x^2+24x\right]_{-6}^2$ OR show correct calculations $224(=\frac{2688}{12})-\frac{307}{12}+\frac{1728}{12}(=144)-\frac{13}{12}$	A1	Accept correct limits also in $\left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{13}{2}x^2+10x\right]_{-6}^2-\left[\frac{7x^2}{2}-14x\right]_{-6}^2$
	$\frac{1024}{3}$	A1	Accept $341\frac{1}{3}$ or exact equiv.

Question	Answer	Marks	Guidance
6(i)	$4\lambda - 3\mu = 24$ $7\lambda - \mu = -9$ $8\lambda - 4\mu = 24$	**M1**	Equate components and form 3 simultaneous eqns. OR $21 + 3\mu = -3 + 4\lambda$ $2 + \mu = 11 + 7\lambda$ $15 + 4\mu = -9 + 8\lambda$
	$4\lambda - 3\mu = 24$ $7\lambda - \mu = -9$	**M1**	Solve any pair of the simultaneous eqns
	$\lambda = -3$, $\mu = -12$	**A1**	Award for λ and μ correct
	$8(-3) - 4(-12) = 24$ so the lines intersect	**B1**	Substitute values into the third eqn and make explicit conclusion e.g. using z component $-9 + 8(-3) = -33$ and $15 + 4(-12) = -33$ Other components compared like this yield -15 and -10 OR substitute correct λ and μ into r_1 and r_2 to get correct intersection point in both.
	$(-15, -10, -33)$ or $\mathbf{r} = -15\mathbf{i} - 10\mathbf{j} - 33\mathbf{k}$	**A1**	
6(ii)	$\theta = \cos^{-1}\dfrac{(4\mathbf{i}+7\mathbf{j}+8\mathbf{k})\bullet(3\mathbf{i}+\mathbf{j}+4\mathbf{k})}{\sqrt{4^2+7^2+8^2}\sqrt{3^2+1^2+4^2}}$	**M1**	Use of correct formula
	$\theta = \cos^{-1}\dfrac{51}{\sqrt{129}\sqrt{26}}$	**A1**	Correct direction vector values used
	28.3°	**A1**	Award also for 0.494

Question	Answer	Marks	Guidance
7	$18x+8y\frac{dy}{dx}-6-4\frac{dy}{dx}=0$	**M1**	Obtain 4 term expression from implicit differentiation but must include $ky\frac{dy}{dx}$.
	$\frac{dy}{dx}=\frac{6-18x}{8y-4}=0$	**M1**	Set their $\frac{dy}{dx}=0$ and solve to obtain a value for x
	$x=\frac{1}{3}$	**A1**	
	$1+4y^2-2-4y=34$	**M1**	Substitute their value for x and obtain a quadratic in y.
	$\left(\frac{1}{3},\frac{7}{2}\right),\left(\frac{1}{3},\frac{-5}{2}\right)$	**A1**	Obtain correct coordinates.
	$\frac{d^2y}{dx^2}=\frac{-18(8y-4)-8\frac{dy}{dx}(6-18x)}{(8y-4)^2}$ $=\frac{-18}{(8y-4)}$	**M1**	A generous attempt at quotient rule, allowing some slips Accept alternative reasons and methods, e.g. one y value is greater than the other with the same x-coordinate and a mix and match of methods involving conics. Do not penalize a description of the ellipse as a circle
	$\frac{d^2y}{dx^2}=-0.75$ at $\left(\frac{1}{3},\frac{7}{2}\right)$ so maximum	**A1**	Final value must be shown
	$\frac{d^2y}{dx^2}=0.75$ at $\left(\frac{1}{3},\frac{-5}{2}\right)$ so minimum	**A1**	Final value must be shown Full credit can be obtained using conics: $9\left(x-\frac{1}{3}\right)^2+4\left(y-\frac{1}{2}\right)^2=36$ M1A1A1 or $(3x-1)^2+(2y-1)^2=36$ M1A1 Ellipse centre $\left(\frac{1}{3},\frac{1}{2}\right)$ $\frac{\left(y-\frac{1}{2}\right)^2}{9}=1$ M1 A1 A1 $y=3.5$ (max) and (min) -2.5

Question	Answer	Marks	Guidance
8(i)	$\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta}$	**M1**	OR $\frac{\mathrm{d}(\tan 3\theta)}{\mathrm{d}\theta} = 3\sec^2 3\theta$ M1 $3\left(1+\tan^2 3\theta\right)$ M1
	$\frac{\mathrm{d}(\tan 3\theta)}{\mathrm{d}\theta} = \frac{3\cos^2 3\theta + 3\sin^2 3\theta}{\cos^2 3\theta}$	**M1**	
	$= 3 + 3\tan^2 3\theta$	**A1**	**Dep** on M marks
8(ii)	$3\int \tan^2 3\theta \mathrm{d}\theta = \tan 3\theta - \int 3\mathrm{d}\theta + c$	**M1**	Some attempt to show evidence for the use of (i) or alternative approaches e.g. $\sec^2 3\theta - 1$: $3\int \tan^2 3\theta \mathrm{d}\theta = 3\int \sec^2 3\theta - \int 3\mathrm{d}\theta + c$
	$\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \tan^2 3\theta \mathrm{d}\theta = \left[\frac{1}{3}\tan 3\theta - \theta\right]_{\frac{\pi}{12}}^{\frac{\pi}{9}}$	**M1**	Award for $a\tan 3\theta + b\theta$. x appearing loses the A1
		A1	For correct integral
	$\left(\frac{1}{3}\tan\frac{\pi}{3} - \frac{\pi}{9}\right) - \left(\frac{1}{3}\tan\frac{\pi}{4} - \frac{\pi}{12}\right)$	**M1**	Award for sight only of substitution of correct limits. May be awarded with incorrect integral i.e. after M0M0
	$\frac{1}{3}(\sqrt{3}-1) - \frac{\pi}{36}$	**A1**	aef
9(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x} - k = 0$	**M1**	Differentiate, put equal to 0 and attempt to solve
	$\Rightarrow x = \frac{1}{k}$	**A1**	aef
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{1}{x^2}$ and at $x = \frac{1}{k}$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -k^2 < 0$ and hence maximum	**B1**	Clear conclusion required. Accept $-\frac{1}{\left(\frac{1}{k}\right)^2} < 0$
	$y = \ln\left(\frac{1}{k}\right) - 1 = -\ln k - 1$	**A1**	aef

Question	Answer	Marks	Guidance
9(ii)	$-\ln k - 1 \geqslant 0$	**M1**	y ordinate stated either = or $\geqslant 0$ M1. M0 if only specific values used Accept a statement that if $y \leqslant 0$ there will be no real roots for B1.
	$k \leqslant \frac{1}{\mathrm{e}}$ AG	**A1**	
9(iii)	$x_{n+1} = x_n - \dfrac{\ln x - \frac{1}{3}x}{\frac{1}{x} - \frac{1}{3}}$	**M1**	Correct formula
	5, 4.570784343, 4.536651853, 4.536403668	**A1**	Obtain at least 2 correct iterates including 5
	4.54	**A1**	Obtain answer correct to 3 s.f.
9(iv)	If $k < 0$, $-kx$ adds a positive quantity onto $\ln x$	**B1**	
	If $k = 0$, the curve is $y = \ln x$ which has no maximum and continuously increases so crosses the x-axis once and has one root	**B1**	Use of graphical arguments, e.g. $x = \mathrm{e}^{kx}$ and intersection points should be convincing Accept If $k = 0$, the curve is $y = \ln x$ which has one root. Arguments leading from $\ln x = 0$ to $x = 1$ should be convincing
	Hence the function $y = \ln x - kx$ is continuously increasing so crosses the x-axis once only.	**B1**	Award if the two cases are considered and an overall conclusion drawn.
9(v)	Should mention a maximum between $0 < k < \frac{1}{\mathrm{e}}$	**B1**	Accept alternative arguments
	Conclude that the curve must cross the x-axis twice giving two real roots.	**B1**	

Question	Answer	Marks	Guidance
10(i)	$\int \frac{2}{y-y^3}\,\mathrm{d}y = \int \mathrm{d}x$	M1	Some intention to separate variables e.g. sight of $\frac{k}{y-y^3} = \frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{k}{y-y^3}\mathrm{d}y$ (k = 1 or 2)
	$\frac{A}{y}+\frac{B}{1+y}+\frac{C}{1-y}$ or $\frac{A}{y}+\frac{By+C}{1-y^2}$ or $\pm\frac{A}{y}+\frac{By+C}{y^2-1}$	M1	Set up partial fractions in a reasonably correct form e.g. ignore sign errors from an attempt to write the $y(y^2-1)$ form
	For example 2, 2, 0 or 1, 1, 0 or 2, –1, 1 or 1, $\frac{1}{2}, -\frac{1}{2}$	A1	Obtain at least one correct value
		A1	Obtain any correct triplet for **their** partial fraction format
	$2\ln y - \ln(1+y) - \ln(1-y) = x + c$ Or $2\ln y - \ln(1-y^2) = x + C$ Or $\ln y - \frac{1}{2}\ln(1-y^2) = \frac{1}{2}x + C$	M1	Obtain $k \ln y \pm m \ln(1+y) \pm n \ln(1-y)$ $= \pm px\ (+c)$ Or $k \ln y \pm m \ln(1-y^2) = \pm px\ (+c)$ Or $k \ln y \pm m \ln(y^2-1) = \pm px\ (+c)$
	$\frac{y^2}{(1+y)(1-y)} = \mathrm{e}^x\mathrm{e}^c$	M1	Use at least one log law correctly
		M1	Obtain an approximately correct expression without logs, e.g. allow a coefficient slip but not omission of $+c$ Accept e^{x+c} This is a fairly strict mark but allow reasonably correct equivalent forms
	$y = \sqrt{\frac{A\mathrm{e}^x}{1+A\mathrm{e}^x}}$	M1	Obtain their reasonable expression in the form $y = \mathrm{f}(x)$ by a reasonable method .
		A1	If the previous mark is M0, M1 cannot be given here Obtain a correct expression for y. Do not accept e^{x+c} for this mark

Question	Answer	Marks	Guidance
10(ii)	As x becomes large and negative Ae^x tends to 0 and $\lim\limits_{x\to-\infty} \mathrm{f}(x)=0$	**B1**	**Dep** (ii) requires a correct y.
	Since $Ae^x \neq 0$ $y=\sqrt{\dfrac{\frac{Ae^x}{Ae^x}}{\frac{1}{Ae^x}+\frac{Ae^x}{Ae^x}}}$	**M1**	**Dep** Accept minimal or no working.
	Hence $\lim\limits_{x\to+\infty} \mathrm{f}(x)=1$	**A1**	