Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Application of Mathematics **October/November 2020**

MARK SCHEME

Maximum Mark: 120

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **20** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	B(5000, 0.00008) oe	**B1**	Binomial and 5000 correct
		B1	0.00008 correct (8×10^{-5} or 1 / 12 500)
1(b)	Large value of n (5000) and small value of p (0.00008)	**B1**	Both, with numerical values referred to (not just "n is large and p is small") May see $n = 5000 > 20$ and $np = 0.4 < 5$. Allow if numerically justified. Showing $\mu \approx \sigma^2$ alone is insufficient.
1(c)	$e^{-0.00008n} > 0.99$	**M1*A1**	M1A0 for $e^{-0.00008n} < 0.01$
	$n < 125.6$	**depM1***	Solving their inequality (could be 57564 or 57565 for M1)
	$n_{max} = 125$	**A1**	

Question	Answer	Marks	Guidance
2(a)	$3\alpha\mu + 4\beta\mu = \mu$	**M1**	Clear use of $E(aX + bY) = aE(X) + bE(Y)$
	$3\alpha + 4\beta = 1$	**A1**	

Question	Answer	Marks	Guidance
2(b)	$\text{Var}(Z) = (7\alpha^2 + 16\beta^2)\sigma^2$	B1	Clear use of $\text{Var}(aX + bY) = a^2\text{Var}X + b^2\text{Var}(Y)$ No need for "Var(Z) =" if intent clear.
	$= (7\alpha^2 + (1-3\alpha)^2)\sigma^2$ $= (16\alpha^2 - 6\alpha + 1)\sigma^2$	M1	Subbing in Var(X) & Var(Y) and either α or β
	$\frac{dV}{d\alpha}$: $32\alpha - 6 = 0$	M1	Or $16\left(\alpha - \frac{3}{16}\right)^2 + \frac{7}{16}$ or equivalent method for minimum
	$\alpha = \frac{3}{16}$ and $\beta = \frac{7}{64}$	A1	
	$\text{Var}(Z) = \frac{7}{16}\sigma^2$	A1	

Question	Answer	Marks	Guidance
3(a)	$e^{-\lambda}\frac{\lambda^{\lambda}}{\lambda!}$	B1	This only
3(b)	N(60, 60)	B1	Stated or implied, allow $\sqrt{60}$ here
	$\Phi\left(\frac{60.5-60}{\sqrt{60}}\right) - \Phi\left(\frac{59.5-60}{\sqrt{60}}\right)$	M1	Or could see use of symmetry. Correct cc needed, allow 60 here.
		A1	Fully correct
	$= 0.525\ldots - 0.474\ldots = 0.0515$	A1	Answer, accept anything in [0.0514, 0.0518] www

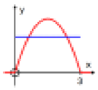
Question	Answer	Marks	Guidance
3(c)	$\frac{1}{\sqrt{\lambda}\sqrt{2\pi}}\mathrm{e}^{0} \approx \mathrm{e}^{-\lambda}\frac{\lambda^{\lambda}}{\lambda!}$	**M1**	Use normal pdf $\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$ with $y = \mu / \lambda$ and $\sigma^2 \approx \mu$. Ignore confusion between μ and λ. (Condone minor slip for M1)
	$\Rightarrow \lambda! \approx \sqrt{2\pi\lambda}\,\mathrm{e}^{-\lambda}\lambda^{\lambda}$ **AG**	**A1**	
3(d)	So $\ln 75! \approx \frac{1}{2}\ln(2\pi \times 75) - 75 + 75\ln(75)$	**M1**	Subbing in $\lambda = 75$ and using law of logs correctly at least once.
	≈ 251.8893 (7 sf)	**A1**	No marks for 'exact' value (251.8904…) 7 sf asked for, but allow fewer provided clearly *not* 251.8904…

Question	Answer	Marks	Guidance
4(a)	$\mathrm{P}(R = r) = pq^{r-1}$ or $p(1-p)^{r-1}$	**B1**	Stated or implied
	$\mathrm{G}(t) = \Sigma \mathrm{P}(R = r)t^{r}$	**M1**	$\Sigma \mathrm{P}(R = r)t^{r}$ used (series could be explicit)
	$= \Sigma pt(qt)^{n-1}$	**A1**	One intermediate step needed
	Using GP formula, $\mathrm{G}(t) = \frac{pt}{1-qt}$ **AG**	**B1**	Mention or quote GP formula or equivalent

Question	Answer	Marks	Guidance
4(b)	Geo(0.3)	**B1**	Stated or implied
	$[\mathrm{G}(t)]^5 = \left(\frac{0.3t}{1-0.7t}\right)^5$	**M1**	Use of $[\mathrm{G}(t)]^5$
	$= 0.3^5 t^5 (1-0.7t)^{-5}$	**M1**	Appropriate form for bin expansion, correct coefficient selected
	Coeff of $t^8 = 0.3^5 \times 0.7^3 \times 35$	**M1**	Use binomial expansion (independent of previous mark)
	= 0.02917…	**A1**	Answer, art 0.0292

Question	Answer	Marks	Guidance
5(a)	$S^2 = \frac{15}{14} \times 5.6^2 = 33.6$	**B1**	$\frac{15}{14}$ used here
	$11.6 \pm 2.977\sqrt{\frac{33.6}{15}}$	**M1**	$11.6 \pm t\sqrt{\frac{S^2}{15}}$, with any t or 2.576
		B1	$t = 2.977$
	= (7.14, 16.06)	**A1**	Both end-points correct to 2dp Not 7.14 < μ < 16.06 unless carefully explained SR no 15/14: 7.30, 15.90 B0M1A1 SR normal: 7.75, 15.46 B1M1A0 SR both: 7.88, 15.33 B0M1A0
	Assume autumn mid-day temperature in London is normally distributed	**B1**	Or equivalent – *not* “sample”. Must be in context.

Question	Answer	Marks	Guidance
5(b)	$\hat{\mu}_{\text{Paris}} = 14.294\ldots$ (243/17)	B1	14.3 or better stated or implied
	$S^2_{\text{Paris}} = \frac{17}{16}\left(\frac{4070}{17} - 14.294\ldots^2\right)$ $\left[= \frac{10141}{272} = 37.283\right]$	M1	Or without $\frac{17}{16}$ [= 10141/289 = 35.090]
	$\hat{\sigma}^2 = \frac{14\times\text{"}33.6\text{"}+16\times\text{"}37.283\text{"}}{15+17-2}$	M1	Find pooled variance estimate or biased with 15, 17
	= 35.6 (35.5643…)	A1	Stated or implied, allow $\frac{S^2_{\text{London}}}{15} + \frac{S^2_{\text{Paris}}}{17}$ but just adding; M0
	$(14.294 - 11.6) \pm 2.042\sqrt{35.5643\left(\frac{1}{15}+\frac{1}{17}\right)}$	M1	Use $\hat{\sigma}^2\left(\frac{1}{15}+\frac{1}{17}\right)$. But 1/32; M0A0. $2.694 \pm 2.042\sqrt{4.46297}$
		B1	2.042 used
	= (–1.62, 7.01)	A1	Both, correct to 2dp (differences may be the other way round)
	Assume both distributions are normally distributed with a common variance	B1	Condone omission of 'both normally distributed' here if final B1 gained in **(a)**. Ignore mention of independence.

Question	Answer	Marks	Guidance
6(a)		B1	Horizontal line $y = ⅓$ in $0 \leqslant x \leqslant 3$ and nothing else (except axis)
		M1	Correct negative parabola, must stop at x-axis but ignore absence of horizontal portions
		A1	Roughly correct relative position (so that areas approx. equal) so must show 2 points of intersection. Ignore y-axis values other than ⅓ which must be correct if shown.

Question	Answer	Marks	Guidance
6(b)	T; more likely to take values close to extremities or more of the distribution of T is further from its mean oe.	B2	T stated with good reason. SC: T stated with dubious or partial reason: B1
6(c)	$\int_0^u \frac{2}{9}x(3-x)\,dx = \left[\frac{1}{3}x^2 - \frac{2}{27}x^3\right]_0^u$	M1	Integration of f(x)
		A1	Correct indefinite integral
	$(F(u) =)\frac{1}{3}u^2 - \frac{2}{27}u^3 = 0.75$	M1	Equation for u, or both $\int_0^{2.02} f(x)\,dx$ and $\int_0^{2.03} f(x)\,dx$ attempted
	$F(2.02) = 0.7496$, $F(2.03) = 0.7540$ or evaluation in equivalent cubic e.g. $g(u) = 8u^3 - 36u^2 + 81$ with explanation	A1	At least one correct evaluation at 2.02 and/or 2.03
	hence $2.02 < u < 2.03$ **AG**	A1	All correct, with justification (could be just inequalities) *Or*:u = 2.0209 or better (2.020945…) A1, hence … A1
6(d)	$F(x) = P(S \leqslant x) = \frac{1}{3}x^2 - \frac{2}{27}x^3$	M1	Obtain CDF
	$P(Y \leqslant y) = P(\sqrt{S} \leqslant y) = P(S \leqslant y^2)$	M1	Use inverse function
	$= F(y^2)$	A1	F(y^2) stated or implied
	$= \begin{cases} 0 & y < 0 \\ \frac{1}{3}y^4 - \frac{2}{27}y^6 & 0 \leqslant y \leqslant \sqrt{3} \\ 1 & y > \sqrt{3} \end{cases}$	A1	$\frac{1}{3}y^4 - \frac{2}{27}y^6$ correct
		B1	0, 1 and all ranges stated and correct (CDF need not be named). Ignore overlap at 0 and/or 1. Condon if single value 0 or 1 missing

Question	Answer	Marks	Guidance
7(a)	Moments about A: $7.5\times1.2=2g\times x\times\sin30°$	M1	Take moments about any sensible point. Correct no, of moments and each must be a force times a distance.
	$x=0.9$ m	A1	Exact answer only, aeef
7(b)	Resolve $\rightarrow$: $X=7.5\cos30°$	M1	Horizontal resolution attempt: allow sin/cos muddle
	$X=6.50$ N	A1	
	Resolve $\uparrow$: $Y+7.5\sin30°=20$	M1	Vertical resolution attempt: 3 forces, allow sin/cos muddle
	$Y=16.25$ N	A1	(Allow 16.2 or 16.3 stated as answer)

Question	Answer	Marks	Guidance
8	$F=0.8R$	M1	$F=0.8R$ stated or used
	$=0.8\times20g\cos15°$ $(=154.548\ldots)$	M1	Resolve perpendicular to slope
	WD against friction $=10\times154.548\ldots=1545$	B1	Correct value for WD, can be implied
	$\Delta\text{KE}=\frac{1}{2}\times20\times3^2=90$	B1	Correct value for change in KE, can be implied
	$\Delta\text{GPE}=-20\times g\times10\sin15°=-517(.638)$	B1	Correct value for ΔGPE, allow wrong/ambiguous sign
	$\text{WD}=1545+90-517.6$	M1	Combine 3 terms
		A1	Signs correct
	$=1118$ J	A1	Answer, art 1120

Question	Answer	Marks	Guidance
8	**Alternative**		
	$F = 0.8R$	**(M1)**	$F = 0.8R$ stated or used
	$= 0.8 \times 20g\cos 15°$ (= 154.548…)	**(M1)**	Resolve perpendicular to slope
	$P - F + 20g\sin 15° = 20a$	**(M1)**	NII parallel to slope with 3 forces, weight resolved and "*ma*".
		(A1)	All correct including signs
	$v^2 = u^2 + 2as \Rightarrow a = 0.45$	**(B1)**	Correct value for acceleration
	$P = 111.8$ soi	**(A1)**	
	WD = 111.8×10	**(M1)**	Use of WD = force × dist
	= 1118 J	**(A1)**	Answer, art 1120

Question	Answer	Marks	Guidance
9(a)	$v = r\omega$	**M1**	
	$= 0.1 \times (2 + 0.866) = 0.2866$ m s^{-1}	**A1**	0.287 or art 0.2866 or better or $\frac{1}{20}\left(4+\sqrt{3}\right)$ aeef

Question	Answer	Marks	Guidance
9(b)	$a_r = r\omega^2$	M1	Use $r\omega^2$
	$= 0.1 \times 2.866^2$ [$= 0.82141\ldots$]	A1	Correct value seen or implied
	$\dot{\omega} = 2\cos(2t)$ [=1]	M1	Find $\dot{\omega}$ and use $r\dot{\omega}$
	$a_t = r\dot{\omega} = 0.1$	A1	Correct value seen or implied
	$\lvert \mathbf{a} \rvert = \sqrt{0.8214^2 + 0.1^2}$	M1	Find magnitude
	$= 0.827(47)$ m s^{-2}	A1	Answer, in range [0.827,0.828]

Question	Answer	Marks	Guidance
10(a)	ΔE: $\frac{1}{2}mu^2 = \frac{1}{2}m \times 1.4^2 + mg \times 0.2$ $(= 0.588 + 1.2 = 1.788)$	M1	Use ΔKE = ΔGPE (= 1.2)
	$u^2 = 5.96$ $(u = 2.441\ldots)$	A1	Stated or implied
	N2 ↑: $T - 0.6g = 0.6 \times \frac{5.96}{0.2} (= 17.88)$	M1	Resolve vertically
	$T = 23.88$ N	A1	Answer art 23.9

Question	Answer	Marks	Guidance
10(b)	Let $\theta°$ be the angle with the upward vertical at Q N2 (towards centre): $mg\cos\theta = m\dfrac{v^2}{0.2}$	M1	Consider radial direction; weight must be resolved (sin or cos)
	ΔE: $\frac{1}{2}mv^2 = \frac{1}{2}m\times1.4^2 - mg\times0.2\cos\theta$	M1	Use ΔKE = ΔGPE; may involve value of u from **(a)** (3 terms)
	$\frac{1}{2}\times0.2mg\cos\theta = \frac{1}{2}m\times1.4^2 - mg\times0.2\cos\theta$	M1	Eliminate v^2 (or solve for v^2 (= 49/75) and sub back in for $\cos\theta$)
	$\cos\theta = 0.32666...$ or $\frac{49}{150}$	A1	Stated or implied
	So direction of velocity: $\cos^{-1}\left(\frac{49}{150}\right) = 70.9°$ to horizontal	A1FT	Correct angle, art 70.9° but must be clearly defined or indicated on diagram. This θ above horizontal or $90 - \theta$ to vertical.
	$v = 0.808(29)\text{ms}^{-1}$	A1	Correct v, art 0.808 or $\frac{7}{15}\sqrt{3}$

Question	Answer	Marks	Guidance
10(c)	$v_y = v\sin\theta$ [= 0.7639…]	M1	Find vertical component of v
	h is given by $v_y^2 = 2gh$	M1	Use *suvat* correctly to find height above Q. If using time must find a time and sub into equation for h for M1.
	$\Rightarrow h = 0.02918\ldots$	A1	Answer, art 0.0292 or better
	Alternative		
	KE at $Q = \frac{1}{2}m \times \frac{49}{75} = \frac{49}{150}m$	(M1)	Could be some PE added in. Could ignore horizontal component.
	Energy at top $= \frac{1}{2}m \times \frac{49}{75}\cos^2\theta + mgh = \left(\frac{49}{150}\right)^3 m + 10mh$ $\frac{49}{150}m = \left(\frac{49}{150}\right)^3 m + 10mh$	(M1)	Finding an expression for KE + PE at top; must be consistent with energy calculation at Q.
	$\Rightarrow h = 0.02918\ldots$	(A1)	Answer, art 0.0292 or better

Question	Answer	Marks	Guidance
11(a)	$\frac{dv}{dt} = 0.1v(12 - v)$	**B1**	Correct differential equation
	$\int \frac{1}{v(12-v)}\,dv = \int 0.1\,dt$	**M1**	Separate and attempt to integrate at least RHS.
		A1	Correct at this stage, 0.1 can be in denominator on left
	$\int \frac{1}{v(12-v)}\,dv = \frac{1}{12}\int \left(\frac{1}{v} + \frac{1}{12-v}\right) dv$	**M1**	Use partial fractions of correct form
		A1	Correct partial fractions
	$\frac{1}{12}\ln\left(\frac{v}{12-v}\right) = 0.1t + c \Rightarrow \frac{v}{12-v} = Ae^{1.2t}$	**A1**	Solve as far as this
	$t = 0,\ v = 4 \Rightarrow A = \frac{1}{2}$	**M1**	Attempt to find c or A $[c = -\frac{1}{12}\ln 2$ or $c = \frac{1}{12}\ln\frac{1}{2}$ oe]
	$2v = 12e^{1.2t} - ve^{1.2t} \Rightarrow 2v + ve^{1.2t} = 12e^{1.2t} \Rightarrow v = \frac{12e^{1.2t}}{e^{1.2t} + 2}$ **AG**	**A1**	Correctly obtain given answer, cwo. Must show collection of v terms.
11(b)	$v = \frac{12}{1 + 2e^{-1.2t}} \to 12\text{ m s}^{-1}$	**B1**	Correct answer; accept without detailed reasoning being shown

Question	Answer	Marks	Guidance
11(c)	$x = \int \frac{12e^{1.2t}}{e^{1.2t}+2}\,dt$	M1	Equate to $\frac{dx}{dt}$ and integrate
	$= 10\ln(e^{1.2t}+2)+c$	A1	
	$x=0,\ t=0 \Rightarrow c=-10\ln 3$	M1	or definite integral with correct limits correctly used
	$x=10\ln\left(\frac{e^{1.2t}+2}{3}\right)$	A1	aeef
	Alternative		
	$v\frac{dv}{dx}=0.1v(12-v) \Rightarrow x=10\int\frac{1}{12-v}\,dv$	(M1)	Use $a=v\frac{dv}{dx}$, rearrange and integrate
	$x=-10\ln(12-v)+c$	(A1)	
	$x=0,\ v=4 \Rightarrow c=10\ln 8$ so $x=10\ln 8-10\ln(12-v)=10\ln 8-10\ln\left(12-\frac{12e^{1.2t}}{e^{1.2t}+2}\right)$	(M1)	Use initial conditions to find c and sub expression for v in aeef
	$x=10\ln\left(\frac{e^{1.2t}+2}{3}\right)$	(A1)	

Question	Answer	Marks	Guidance
12(a)	$\frac{2.5}{0.5}e = 0.2 \times g$	M1	Use $\frac{\lambda}{l}x = mg$
	$e = 0.4$ (m)	A1	Answer 0.4 or aeef
12(b)	$0.05 \times 5.3 = 0.25u$	M1	Use conservation of momentum
	$u = 1.06$ (m s^{-1})	A1	
12(c)	New equilibrium extension e' is given by $\frac{2.5}{0.5}e' = 0.25 \times g$	M1	Find new equilibrium position
	$e' = 0.5$	A1	Stated or implied (can be recovered)
	Let general extension below new equilibrium position be x $2.5 - \frac{2.5}{0.5}(e' + x) = 0.25\ddot{x}$ www	M1	New equation of motion including their e' and x
		A1	All correct including signs
	$\ddot{x} = -20x$ which is SHM (www)	A1	Correct equation and SHM stated SR: e' not considered: B1 for '$\ddot{x} = -20x$ which is SHM'

Question	Answer	Marks	Guidance
12(c)	**Alternative 1**		
	Let total extension of string be x: $2.5-\frac{2.5x}{0.5}=0.25\ddot{x}$	**(M1)**	NII with 2 forces, *ma* and correct signs
	$\ddot{x}=-20(x-0.5)$	**(A1)**	Correct and rearranged to useful form
	$y=x-0.5\Rightarrow\ddot{y}=\ddot{x}$	**(M1)**	Stated or implied
	$\Rightarrow\ddot{y}=-20y$ which is SHM	**(A1)**	Correct equation and SHM stated
	with a new equilibrium position 1m below the ceiling oe	**(A1)**	
	Alternative 2		
	New equilibrium extension e' is given by $\frac{2.5}{0.5}e'=0.25\times g$	**(M1)**	
	$e'=0.5$	**(A1)**	
	Let general extension below new equilibrium position be x $0.25\ddot{x}=\pm\left(0.25g-\frac{2.5(x+0.5)}{0.5}\right)$ or $\pm\left(\frac{2.5(0.5-x)}{0.5}-0.25g\right)$	**(M1)**	
		(A1)	If M1M0 then SC1 for $\ddot{x}=-20x$ or $-(2\sqrt{5})^2x$ seen
	$\Rightarrow\ddot{x}=-20x$ (www) which is SHM	**(A1)**	

Question	Answer	Marks	Guidance
12(c)	**Alternative 3**		
	Let total extension of string be x m and consider energy with eg x m below ceiling at 0 PE level Total energy $=\frac{1}{2}\times 0.25v^2+\frac{2.5x^2}{2\times 0.5}-0.25gx=\text{const}$	(M1)	3 forms of energy added and equated to constant
	$0.25v\frac{\mathrm{d}v}{\mathrm{d}x}+5x-2.5=0 \Rightarrow \ddot{x}+20x-10=0$	(M1)	Differentiating both sides wrt x and using $v\frac{\mathrm{d}v}{\mathrm{d}x}=\ddot{x}$
	$y=x-0.5 \Rightarrow \ddot{y}=\ddot{x}$	(M1)	Stated or implied
	$\Rightarrow \ddot{y}=-20y$ which is SHM	(A1)	Correct equation and SHM stated
	with a new equilibrium position 1m below the ceiling oe	(A1)	
12(d)	Amplitude a is given by $v^2=\omega^2(a^2-x^2)$	M1	Use of formula with numerical v, ω and x to calculate a
	$1.06^2=20(a^2-0.1^2)$	A1	Correct $\lvert x \rvert$ used, with their 1.06 from **(b)** and their 20 from **(c)**
	$a = 0.257$ m	A1	
	Alternative 1		
	Suppose 0 PE level is at bottom Energy just after collision $=\frac{1}{2}\times 0.25\times 1.06^2+0.25g(0.1+a)+\frac{2.5\times 0.4^2}{2\times 0.5}$	(M1)	$0.79045 + 2.5a$ but this depends on where 0 PE level is set
	Energy at bottom $=\frac{2.5\times(0.5+a)^2}{2\times 0.5}$ = Energy after collision	(M1)	$2.5a^2 + 2.5a + 0.625$
	$2.5a^2 = 016545 \Rightarrow a = 0.257$ m	(A1)	

Question	Answer	Marks	Guidance
12(d)	**Alternative 2**		
	$x = a\cos\left(\sqrt{20}t + \phi\right)$ and $t = 0$, $x = -0.1 \Rightarrow a\cos\phi = -\frac{1}{10}$ $v = -\sqrt{20}a\sin\left(\sqrt{20}t + \phi\right)$ and $t = 0$, $v = 1.06$	**(M1)**	Using general form for x in terms of t at $t = 0$
	$\Rightarrow a\sin\phi = -\frac{1.06}{\sqrt{20}}$	**(M1)**	Differentiating to find v in terms of t and using $t = 0$
	$\cos^2\phi + \sin^2\phi = 1 \Rightarrow a^2 = \frac{1}{100} + \frac{1.06^2}{20}$ $\Rightarrow a = 0.257$ m	**(A1)**	