Cambridge
O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

 Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1	$x = 1$	**B1**	
	$-3x - 2 = x + 4$ oe	**M1**	
	$x = -1.5$ oe	**A1**	
2(i)	$\dfrac{\frac{1}{\sin} - \frac{\cos x}{\sin x}}{1 - \cos x}$	**M1**	express in terms of sin$x$ and cos$x$
	$\dfrac{(1 - \cos x)}{\sin(1 - \cos x)}$	**A1**	rewrite not as a fraction within a fraction
	$\dfrac{1}{\sin x} = \operatorname{cosec} x$	**A1**	correct completion **answer given**
2(ii)	$\left[\sin x = \dfrac{1}{2}\right]$ $x = 30°$	**B1**	
	$x = 150°$ nfww	**B1**	no extra answers
3	$(1 + ax)^5 = 1 + 5ax + 10a^2x^2 + 10a^3x^3$ soi	**B1**	4 terms not ${}^nC_r$ notation
	$[2] + (10a + b)x + (5ab + 20a^2)x^2$	**M1**	obtain expansion with 2 terms in x, 2 terms in x^2
	equate terms in x and x^2 to give two equations in a and b each consisting of three terms	**M1**	
	$10a + b = 32$ $5ab + 20a^2 = 210$	**A1**	correct equations imply previous two M marks
	eliminate b	**M1**	
	obtain $3a^2 - 16a + 21 = 0$ correctly	**A1**	**answer given**
	$a = 3$ and $b = 2$	**B1**	
	$c = 720$ only	**B1**	no additional answers
4(i)	$y = 2(x - 1)^2 - 9$	**B3**	$a = 2$, $b = 1$, $c = -9$ in correct form. **B1** for each
4(ii)	minimum *their* -9	**B1**	**FT** from *their* correct form, with $a > 0$
	when $x =$ *their* 1	**B1**	**FT** from *their* correct form, with $a > 0$

Question	Answer	Marks	Guidance
4(iii)	$x=\sqrt{p}$ or $p=x^2$ soi	**B1**	
	$(x-1)=\sqrt{\frac{9}{2}}$ or $\left(\sqrt{p}-1\right)=\sqrt{\frac{9}{2}}$ oe	**M1**	$(x-b)=\sqrt{\frac{-c}{a}}$ $\left(\sqrt{p}-b\right)=\sqrt{\frac{-c}{a}}$ using *their* values of a, b, c from (i)
	$p=9.74$	**A1**	completion not involving use of quadratic formula
5(a)	$\tan\left(y-\frac{\pi}{4}\right)=(\pm)\sqrt{3}$	**M1**	± 1.73…
	$y-\frac{\pi}{4}=\frac{\pi}{3}$ or $\frac{2\pi}{3}$	**A1**	1.04(7…) or 2.09(4…)
	$y=\frac{7\pi}{12}$ or 1.83	**A1**	
	$y=\frac{11\pi}{12}$ or 2.88	**A1**	
5(b)	correctly rewrite equation in terms of sin$z$ and cos$z$	**M1**	
	use $\sin^2 z = 1 - \cos^2 z$	**M1**	appropriate use of Pythagorean identity for forming an equation in one trig ratio
	$6\cos^2 z - 7\cos z + 1 = 0$ **oe**	**A1**	
	$(6\cos z - 1)(\cos z - 1) = 0$	**M1**	solve three term quadratic in cos$z$
	80.4°	**A1**	
	279.6°	**A1**	
6(i)	$[\tan ACB =]\ \frac{3+\sqrt{3}}{3-\sqrt{3}}$	**B1**	
	rationalise with $3+\sqrt{3}$	**M1**	
	simplify showing at least 3 terms in numerator to $2+\sqrt{3}$	**A1**	
6(ii)	$(AC)^2=\left(3+\sqrt{3}\right)^2+\left(3-\sqrt{3}\right)^2$ oe	**M1**	Pythagoras
	at least 4 terms $12+6\sqrt{3}+12-6\sqrt{3}$	**A1**	

Question	Answer	Marks	Guidance
	$AC = 2\sqrt{6}$	A1	
7(i)	evidence of differentiation $(3x+2)^{-3}$	M1	
	$-12(3x+2)^{-3} \times 3$	A1	may use PR or QR on fraction part
	$+1$	B1	
	set *their* $\frac{dy}{dx} = 0$	M1	$1 - 36(3x+2)^{-3} = 0$
	$x = 0.43$ nfww	A1	
	$y = 0.98$ only	A1	
7(ii)	$\frac{-2}{3x+2}$ oe	B1	
	$\frac{1}{2}x^2$	B1	
	$\left[\frac{-2}{6+2}+2\right]-\left[\frac{-2}{2}\right]$	M1	insert correct limits into *their* two term integral and subtract two non-zero terms in correct order
	2.75 nfww	A1	2.75 following B1 B1implies M1
8(i)	$p = -4$	B1	
8(ii)	$(x-2)(x-3)(x+4)$	M1	**FT** $(x-2)(x-3)(x-p)$
	$(x^2-5x+6)(x+4)$	A1	**FT** $(x^2-5x+6)(x-p)$ multiply out two factors
	correctly obtain $a = -1$ $x^3 - x^2 - 14x + 24$	A1	**answer given**
	$b = -14$ stated	B1	
8(iii)	$\frac{dy}{dx} = 3x^2 - 2x - 14$	B1	**FT** *their* numerical b $3x^2 - 2x + b$
8(iv)	set *their* $\frac{dy}{dx}$ equal to 2	M1	**FT** *their* numerical b
	$x = 2$	A1	
	$y = 40$ only	A1	no additional answers
8(v)	$y - 40 = 2(x+2)$ $(y = 2x + 44)$	B1	
9(i)	$\overrightarrow{AD} = 2\mathbf{a} + \mathbf{b}$	B1	

Question	Answer	Marks	Guidance
	$\overrightarrow{OX} = \mathbf{a} + \lambda(2\mathbf{a} + \mathbf{b})$	B1	
9(ii)	$\overrightarrow{BC} = 3\mathbf{a} - 2\mathbf{b}$	B1	
	$\overrightarrow{OX} = 2\mathbf{b} + \mu(3\mathbf{a} - 2\mathbf{b})$	B1	
9(iii)	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate for **a** or **b**	M1	
	$1 + 2\lambda = 3\mu$ and $\lambda = 2 - 2\mu$	A1	
	solve correct equations for λ or μ	M1	
	$\lambda = \frac{4}{7}$ and $\mu = \frac{5}{7}$	A1	
9(iv)	$\frac{4}{3}$ or 4 : 3	B1	FT $\lambda/(1-\lambda)$ $0 < \lambda < 1$
10(i)	$\mathrm{gf}(x) = \mathrm{e}^{2(\ln(3x+2))} - 4$	B1	
	their gf = 5	M1	
	use $\ln a^p = p\ln a$ or $\mathrm{e}^{\ln a} = a$ or $\ln \mathrm{e}^a = a$	B1	correct use of log/exponential relationship seen anywhere
	$3x + 2 = 3$ or $(3x + 2)^2 = 9$	A1	3 may take the form of $\mathrm{e}^{0.5\ln 9}$ 9 may take the form of $\mathrm{e}^{\ln 9}$
	$x = \frac{1}{3}$ only	A1	
10(ii)	$x = \dfrac{\mathrm{e}^y - 2}{3}$	M1	find x in terms of y
	$\dfrac{\mathrm{e}^x - 2}{3}$ $\left(= \mathrm{f}^{-1}(x) \text{ or } = y\right)$	A1	interchange x and y correct completion
10(iii)	$\dfrac{\mathrm{e}^x - 2}{3} = \mathrm{e}^{2x} - 4$	M1	*their* $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$
	$3\mathrm{e}^{2x} - \mathrm{e}^x - 10$ $(= 0)$	A1	obtain quadratic in e^x must be arranged as a three term quadratic in order shown
	$(3\mathrm{e}^x + 5)(\mathrm{e}^x - 2)$ $(= 0)$	M1	solve for e^x
	$x = \ln 2$ or 0.693 only	A1	